Week 2 (due April 12)

1. Consider scalar field theory in $D = 3$ space-time dimensions with the Lagrangian

$$L = \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - \frac{m^2}{2}\phi^2 - \frac{\lambda}{k!}\phi^k.$$

Here k is an integer. Express the superficial degree of divergence of a Feynman diagram in terms of the number of external legs and the number of vertices. For which values of k is the superficial degree of divergence independent of the number of vertices? (This value corresponds to a renormalizable theory). For which values of k does the degree of divergence increase with the number of vertices? (This corresponds to a nonrenormalizable theory).

2. Consider scalar field theory in $D = 4$ spacetime dimensions and the Lagrangian

$$L = \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - \frac{m^2}{2}\phi^2 - \frac{\lambda}{3!}\phi^3.$$

(a) Compute the superficial degree of divergence of diagrams in this theory. Show that only a finite number of diagrams have nonnegative superficial degree of divergence and draw all such diagrams.

(b) Compute the 2-point Green's function in this theory to 1-loop accuracy. Use dimensional regularization and Modified Minimal Subtraction ($\overline{MS}$). Express the physical mass in terms of m (to 1-loop accuracy).

(c) Compute the 3-point Green's function in this theory to 1-loop accuracy. (In order to reduce the loop integral to the standard form, you will need to put three propagators together. This can be accomplished by means of the Feynman formula eq. (6.41) in Peskin-Schroeder).