Week 2 (due April 16)

1. In three space-time dimensions, there exists an alternative gauge-invariant action for a gauge field A_μ:

$$S_{CS} = k \int d^3x \ \epsilon^{\mu\nu\rho} A_\mu \partial_\nu A_\rho.$$

Here $\epsilon^{\mu\nu\rho}$ is the completely antisymmetric tensor. The action is known as the Chern-Simons action.

(a) Show that although the Chern-Simons Lagrangian is not gauge-invariant, the action is. Further, show that any solution of the equations of motion is gauge-equivalent to a trivial solution. Thus the theory does not have any physical degrees of freedom.

(b) Now consider a gauge-invariant action which is the sum of the Maxwell and Chern-Simons actions:

$$S = -\frac{1}{4e^2} \int d^3x F_{\mu\nu} F^{\mu\nu} + k \int d^3x \ \epsilon^{\mu\nu\rho} A_\mu \partial_\nu A_\rho.$$

This theory is called a topologically massive gauge theory. Write down the equations of motion and deduce from them that each component of $F_{\mu\nu}$ satisfies the massive Klein-Gordon equation, for some mass m.

(c) Perform the canonical quantization of the theory in part (b) and show that it describes massive noninteracting bosonic particles with spin 1 and a single polarization state.

(d) Find the propagator of the theory in parts (bc).