Week 4 (due April 26)

1. Consider QED in three-dimensional space-time. The Lagrangian of this theory has exactly the same form as in four dimensions. Note that 3d Dirac gamma-matrices can be taken to be $\gamma^0 = i\sigma_1, \gamma^1 = \sigma_2, \gamma^2 = \sigma_3$, where $\sigma_i, i = 1, 2, 3$ are the Pauli matrices. That is, spinors are two-component.

(a) Compute the traces of up to four gamma-matrices in three-dimensional space-time and express the results in terms of the Minkowski metric $g^{\mu\nu}$ and the completely anti-symmetric tensor $\epsilon^{\mu\nu\rho}$ with $\epsilon^{012} = 1$. What is the analogue of γ_5?

(b) Compute the superficial degree of divergence of Feynman diagrams in this theory as a function of the number of external legs and the number of vertices. Is this theory genuinely renormalizable, nonrenormalizable, or superrenormalizable? What is the engineering dimension of the electric charge in this theory?

(c) Show that both the vacuum polarization and the electron self-energy are actually convergent at one loop. Determine the most general form of the vacuum polarization based on Lorenz symmetry and gauge invariance. (Caution: in three dimensions, electron mass breaks parity invariance, so one cannot assume parity-invariance when writing down the general form for $\Pi_{\mu\nu}(q)$.) Based on the results of parts (b) and (c), which counterterms does one need to renormalize 3d QED?

2. Problem 7.3(a) in Peskin and Schroeder.