Week 5 (due May 3)

1. Consider scalar QED (see Problem 1, Week 10, winter quarter, for the precise Lagrangian).

(a) Compute the superficial degree of divergence of a Feynman diagram in terms of the number of scalar and photon external legs and the number of vertices. Don't forget that in this theory there are two different kinds of vertices (the triple vertex and the "sea-gull" vertex). Do you expect the theory to be renormalizable or nonrenormalizable?

(b) Classify all superficially divergent diagrams. Can one absorb these divergences into renormalization of the charge and the fields ϕ and A_μ? If not, what other counterterm in the Lagrangian does one need? How many coupling constants does the quantum Lagrangian (i.e. the Lagrangian including the counterms) depend on?

(c) Compute vacuum polarization (i.e. photon self-energy) in scalar QED at one loop. Compute the effective momentum-scale dependent electric charge (or more precisely, e^2), the way we did it in class for ordinary QED, and find its behavior at short distances (i.e. for $-q^2 \gg m^2$). Does effective coupling $e^2(q^2)$ become larger or smaller at short distances?