Week 7 (due May 17)

1. Problem 12.2 in Peskin and Schroeder. Note that 2d Dirac algebra and spinors are discussed in problem 11.3.

2. (a) Show by a direct computation that under a gauge transformation

$$A_\mu(x) \mapsto U(x)A_\mu(x)U^{-1}(x) + iU(x)\partial_\mu U^{-1}(x)$$

the gauge field strength $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu]$ transforms as

$$F_{\mu\nu} \mapsto U F_{\mu\nu} U^{-1}.$$

(b) The YM Lagrangian is proportional to a gauge-invariant expression $\mathrm{Tr} F_{\mu\nu}F^{\mu\nu}$. Another possible gauge-invariant expression of the same engineering dimension is

$$\epsilon^{\mu\nu\rho\sigma}\mathrm{Tr} F_{\mu\nu}F_{\rho\sigma}$$

In the case of the gauge group $U(1)$, this expression can be written as a total derivative, and is usually ignored (being a total derivative, it does not affect the Euler-Lagrange equations of motion). Show that the same is true in the Yang-Mills theory, i.e. the expression is a total derivative and therefore does not affect the equations of motion. (It is important in the quantum theory though).