Week 10 (due March 15)

1. Consider scalar QED, i.e. a theory with a Lagrangian

$$L = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + (D_\mu\phi)^\dagger D^\mu\phi - m^2\phi^\dagger\phi,$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the field-strength of electromagnetic field, ϕ is a complex scalar field, and $D_\mu\phi = \partial_\mu\phi - ieA_\mu\phi$.

(a) Compute the amplitude for Compton scattering in scalar QED to leading order in perturbation theory. (Do not forget about the sea-gull vertex coming from the term $e^2\phi^\dagger\phi A_\mu A^\mu$ in the Lagrangian; because of this vertex there is one extra Feynman diagram compared to ordinary QED.)

(b) We can write the amplitude as $\epsilon_\mu\epsilon'^*_\nu\mathcal{M}^{\mu\nu}$, where ϵ_μ and ϵ'_ν are polarization vectors of initial and final photons. Show that $\mathcal{M}^{\mu\nu}k_\mu = 0$ and $\mathcal{M}^{\mu\nu}k'_\nu = 0$, where k and k' are 4-momenta of initial and final photons, respectively.

(c) Compute the differential and total cross-section for Compton scattering in scalar QED. Compare with the corresponding formula in ordinary QED. Do they agree in the limit where the energy of the photon is much smaller than the mass of the electron (Thomson scattering)?