Week 5 (due Feb. 8)

1. (a) Consider the following QM Hamiltonian:

$$H = \frac{1}{2}p^2 + \frac{1}{2}\omega^2 q^2.$$

It describes harmonic oscillator with frequency ω.

Compute the "propagator"

$$G(t - t') = \langle 0|T(q(t)q(t'))|0\rangle,$$

where $q(t)$ is the operator in the Heisenberg picture. To this end, express $q(t)$ in terms of creation-annihilation operators and then evaluate the vacuum expectation value.

(b) Show that the Fourier transform of $G(t)$ is

$$\tilde{G}(E) = \lim_{\epsilon \to 0+} \frac{i}{E^2 - \omega^2 + i\epsilon}.$$

2. Let A be a positive-definite Hermitian matrix of size $n \times n$ and B, C be complex vectors in $\mathbb{C}^n$. Let z be a variable taking values in $\mathbb{C}^n$. Show that the integral over $\mathbb{C}^n$

$$I(A, B, C) = \int \prod_{i=1}^{n} dz_i d\bar{z}_i \exp\left(-z^\dagger A z - B^\dagger z - z^\dagger C\right)$$

is equal to

$$D_n (\det A)^{-1} \exp\left(B^\dagger A^{-1} C\right)$$

where D_n is a number which depends only on n. Determine D_n. Show that the exponential factor in the answer can be obtained by the following procedure: one extremizes the expression

$$-z^\dagger A z - B^\dagger z - z^\dagger C$$

with respect to z and $z^\dagger$ treating them as independent complex vectors, and then substitutes the values of z and $z^\dagger$ corresponding to the extremum into the above expression.