Week 5 (due Feb. 13)

Reading: Srednicki, sections 39, 23, 40.

1. (30pts) Problem 39.4.

2. Problem 40.1.

3. Consider a theory of N Weyl fermions χ^i, $i = 1, \ldots, N$. The most general quadratic Hermitian Lorenz-invariant Lagrangian of first order in derivatives is

$$L = i\chi_i^\dagger \bar{\sigma}^\mu \partial_\mu \chi^i - m_{ij}\chi^i\chi^j - m_{ji}^*\chi^{\dagger i}\chi^{\dagger j}.$$

(a) Show that without loss of generality the matrix m_{ij} can be taken to be symmetric. Show that this Lagrangian describes a theory of N independent massive Majorana particles. What are their masses? (Hint: an arbitrary complex symmetric matrix M can be "diagonalized", i.e. one can find a unitary matrix U such that $U^t M U = D$ is diagonal and the diagonal entries are nonnegative real numbers.)

(b) If N is even, one can arbitrarily pair up Weyl spinors χ^i into $k = N/2$ Dirac spinors Ψ^p, $p = 1, \ldots, k$. Rewrite the above Lagrangian in terms of Ψ. Note that the mass terms in this new Lagrangian are of two kinds: Dirac-type terms of the form $A_{pq}\bar{\Psi}^p\Psi^q$ and Majorana-type mass terms.

(c) What are the continuous symmetries of this theory for generic m_{ij}? Show that for generic m_{ij} the theory is invariant under suitably defined parity and time-reversal symmetries. How do these symmetry transformations act on the fields χ^i?