Week 9 (due March 8)

1. Let n^μ be a time-like unit 4-vector, i.e. $n_\mu n^\mu = 1$. A covariantized version of the Coulomb gauge is to require $n^\mu A_\mu = 0$ and $\partial^\mu A_\mu = 0$. If we choose a frame where $n^\mu = (1, 0, 0, 0)$, these gauge conditions reduce to $A_0 = 0, \partial_i A_i = 0$.

In this "covariantized" Coulomb gauge, the photon polarization vectors satisfy $n^\mu \epsilon_\mu(k) = 0$, $k^\mu \epsilon_\mu = 0$. Hence the polarization sum

$$\Pi_{\mu\nu}(k) = \sum_{i=1,2} \epsilon_\mu(k)\epsilon^*_\nu(k)$$

satisfies

$$\Pi_{\mu\nu} k^\mu = 0, \quad \Pi_{\mu\nu} n^\mu = 0, \quad \Pi_{\mu\nu} \eta^{\mu\nu} = -2.$$

Using these three properties, express $\Pi_{\mu\nu}(k)$ in terms of 4-vectors k and n. (Hint: write down the most general symmetric tensor which depends on k and n and determine the coefficients by imposing the above three conditions).

2. Use the Faddeev-Popov method to derive the photon propagator in the Coulomb gauge $\partial_i A_i = 0$. In particular, show that the propagator for A_0 computed via the path-integral does not vanish. This is contrary to naive expectations, according to which $\partial_i A_i = 0$ together with Maxwell equations imply that A_0 satisfies the Laplace equation, and hence A_0 vanishes identically. What do you get if you act on the 2-point function

$$\langle 0|T(A_0(x)A_0(y))|0\rangle$$

with the 3-dimensional Laplace operator $\Delta = \partial^i \partial_i$?

3. In class we have written down the amplitude for Compton scattering at leading order in perturbation theory. It is proportional to polarization vectors $\epsilon(k)$ and $\epsilon'(k')$ of incoming and outgoing photons and therefore can be written as

$$\epsilon_\mu(k)\epsilon^{'*}(k')_\nu \mathcal{M}^{\mu\nu}.$$

Show by an explicit computation that the tensor $\mathcal{M}^{\mu\nu}$ satisfies

$$k_\mu \mathcal{M}^{\mu\nu} = 0, \quad k'_\nu \mathcal{M}^{\mu\nu} = 0.$$

(Hint: the spinor wavefunction $u(p)$ satisfies $(\not{p} - m)u(p) = 0$. You will also need the conservation law $p + k = p' + k'$.)