Fall quarter, week 1 (due Oct. 10)

1. Bring the thumb and the index finger of your left hand together so their tips touch and form a ring. Do the same with your right hand. Now your fingers form two closed rings. This is configuration A. nfiguration B is the same, but the two rings are linked. Is it possible to continuously deform B into A without breaking the rings? Assume your body is like that of an amoeba and can be deformed arbitrarily. No tearing is allowed.

2. Prove that a closed subset of a compact topological space is compact.

3. If Y is a set, then the diagonal subset of $Y \times Y$ consists of the points of the form (y, y), where $y \in Y$ is arbitrary.

(a) Let Y be a Hausdorff space. Prove that the diagonal subset is closed. Further, let f, g be continuous maps from a topological space X to a Hausdorff topological space Y. Show that the set of points $x \in X$ such that $f(x) = g(x)$ is closed.

(b) Show that if two continuous maps $f, g : X \to Y$, where Y is Hausdorff, agree on a dense subset, then they agree everywhere.

4. Prove that a topological space X is connected if and only if any continuous map from X to a discrete topological space Y is constant. (Here by a discrete topological space I mean a set with a discrete topology).