Fall quarter, week 2 (due Oct. 17)

1. We can identify S^3 with a point removed with $\mathbb{R}^3$ (for example, via the stereographic projection). Then all but one of the fibers of the Hopf fibration $p : S^3 \to S^2$ turn into circles in $\mathbb{R}^3$. The remaining circle turns into a line. Sketch these circles and show that any two of them are linked like rings on a keychain.

2. Using the exact sequence of a fibration, prove that $\pi_3(S^2) = \mathbb{Z}$ and $\pi_2(\mathbb{CP}^n) = \mathbb{Z}$ for all $n > 0$.

3. Suppose $n > 1$. What is the next non-trivial homotopy group of $\mathbb{CP}^n$ after π_2?

4. Let $p : E \to X$ be a covering. It was stated in class that the map of fundamental groups $p_* : \pi_1(E) \to \pi_1(X)$ is injective. Show that this follows from the exact sequence of a fibration. Use this fact to answer the following question: can a sphere with g handles ($g > 1$) cover a torus? That is, does there exist a covering $p : E \to X$ where E is a sphere with g handles and X is a torus T^2?