Fall quarter, week 3 (due Oct. 24)

1. Compute the homology of the Klein bottle (with integer coefficients) using your favorite cell decomposition (it is easiest to think of the Klein bottle as a square with some sides identified).

2. Consider a hexagon with vertices $A_1, \ldots, A_6$ (in this cyclic order). Identify the following pairs of sides: A_0A_1 and A_1A_2, A_2A_3 and A_3A_4, A_4A_5 and A_5A_0 (with this orientation). Compute the homology of the resulting surface.

3. A sphere of dimension n has a cell decomposition with only two cells, one of dimension n and one of dimension 0. An alternative cell decomposition involves two cell in every dimension from 0 to n. It is constructed as follows: one decomposes S^n into two open hemispheres and a sphere of dimension $n-1$ (the equator). The two hemispheres are n-cells. Then one repeats the process with the equatorial S^{n-1}. The advantage of this cell decomposition is that the antipodal map just exchanges the two cells of each dimension. Therefore one gets a cell decomposition of $\mathbb{RP}^n$ with one cell in each dimension from 0 to n. Use this cell decomposition to compute the homology of $\mathbb{RP}^n$ with integer coefficients.

4. Use the same cell decomposition as in problem 3 to compute the homology of $\mathbb{RP}^n$ with coefficients in $U(1) = \mathbb{R}/\mathbb{Z}$ and in $\mathbb{Z}_m$ where m is an arbitrary positive integer.