Week 2 (due April 15)

1. Consider a surface S in $\mathbb{R}^3$ given by the equation $z = f(x, y)$. The standard flat metric on $\mathbb{R}^3$ induces a curved metric on this surface. It also gives rise to a second fundamental form.

(a) Express the metric of S at a point (x, y) in terms of $f(x, y)$ and its derivatives.

(b) Express the second fundamental form of S in terms of $f(x, y)$.

2. Show that the isomorphism of the Lie algebras of $SO(n)$ and $Spin(n)$ maps an antisymmetric matrix a_{ij} (regarded as an element of the Lie algebra of $SO(n)$) to

$$\frac{1}{4} \sum_{i,j} a_{ij} e_i \circ e_j,$$

where e_i is an element of an orthonormal basis of $\mathbb{R}^n$, regarded as an element of $Cl(n)$.