Week 1 (due Jan. 14)

All numbered problems are from Morita.

1. Problem 3.8.

2. For a noncompact manifold M of dimension n, one can define a new cohomology theory as follows. Let $\Omega_c^k(M)$ the space of C^∞ k-forms with compact support. The exterior differential d makes the graded vector space

$$\Omega_c = \oplus_{k=0}^n \Omega_c^k(M)$$

into a cochain complex. Its cohomology $H_c^k(M)$, $k = 0, \ldots, n$ is called the de Rham cohomology with compact support. Compute $H_c^k(M)$ for $M = \mathbb{R}$ for all k. (Warning: de Rham cohomology with compact support is not homotopy-invariant, so in this computation one cannot replace $\mathbb{R}$ with a point.)

3. Compute Cech cohomology of S^1 directly from the definition.