Week 2 (due Jan. 21)

1. Show among all closed surfaces only T^2 can cover T^2.
2. Classify all possible coverings of the form $T^2 \to T^2$.
3. Recall that the Euler characteristic of a topological space X is the sum

$$\chi(X) = \sum_i (-1)^i b_i(X),$$

where $b_i(X)$ is the i^{th} Betti number of X. One can show show that if $p : Y \to X$ is an n-sheeted cover of X, then $\chi(Y) = n\chi(X)$.

3(a). Let $\mathbb{R}P^2$ be the projective plane, i.e. S^2 with antipodal points identified. Let K be the Klein bottle. Can K cover $\mathbb{R}P^2$? Can $\mathbb{R}P^2$ cover K?

3(b). Which closed surfaces can be covered by S^2?