Week 6 (due Feb. 18)

1. Problem 6.8 in Morita.

2. Consider a unit sphere S^2 in $\mathbb{R}^3$. The tangent bundle to $\mathbb{R}^3$ is trivial and one can define a connection on it by letting

$$\nabla \frac{\partial}{\partial x^i} = 0,$$

where x^1, x^2, x^3 are Cartesian coordinates on $\mathbb{R}^3$. The restriction of $T\mathbb{R}^3$ to the sphere has the tangent bundle of S^2 as a subbundle. Thus we may define a connection on TS^2 using the orthogonal projection method described in class.

(a) If we use the standard spherical coordinates θ, ϕ on the unit sphere, TS^2 is spanned by $\frac{\partial}{\partial\theta}$, $\frac{\partial}{\partial\phi}$. Express these tangent vectors in terms of $\frac{\partial}{\partial x^i}$. Show that they are orthogonal but not normalized correctly. Find the correct normalization and thereby get an orthonormal trivialization of TS^2. (Note: spherical coordinates are good away from the poles only, so this does not give a global trivialization of TS^2 which is in fact nontrivial.)

(b) Compute the connection 1-forms for TS^2 with respect to the above trivialization of TS^2. Also compute the curvature tensor.

(c) Compute the connection 1-form and curvature for TS^2 with respect to the orthogonal but not orthonormal trivialization $\frac{\partial}{\partial\theta}$, $\frac{\partial}{\partial\phi}$.