Week 1 (due Oct. 6)

1. A Laurent polynomial in one variable x is an expression of the form

$$\sum_{n=-N}^{N} c_n x^n,$$

where N is a positive integer, and $c_n, n = 1, \ldots, N$, are complex numbers. It is easy to see that Laurent polynomials form an algebra (i.e. the set of Laurent polynomials is closed under taking linear combinations and products). Exhibit a presentation of this algebra using two generators and the smallest possible number of relations.

2. Bring the thumb and the index finger on your left hand together, so that their tips touch. Do the same with the fingers on your right hand. This is configuration A. Note that you fingers form two closed rings. Configuration B is the same as A, except the two rings are linked. Is it possible to unlink the two rings, i.e. deform B into A, without breaking them? Assume that your body is like amoeba's, i.e. its shape can be deformed arbitrarily, but no tearing is allowed. (In this problem, rigor is unimportant.)

3. A *finite graph* is a figure consisting of a finite number of points (vertices) connected by a finite number of arcs (edges). The *degree* of a vertex is the number of edges issuing from it. A graph is called *Eulerian* if it can be drawn in a single sweep, without going over any edge twice.

a. Show that the number of vertices of odd degree is always even.

b. Show that if all vertices of a connected finite graph have even degree, then the graph is Eulerian, and it can be drawn in one sweep starting at any point.

c. Show that a connected finite graph is Eulerian iff it contains at most two vertices of odd degree. Use this fact to solve the Euler's bridge problem.

4. Prove that the standard $\epsilon - \delta$ definition of continuity for functions from $\mathbb{R}^n$ to $\mathbb{R}^m$ is equivalent to the topological one.

5. Give an example of a map between topological spaces which is continuous, one-to-one, but is not a homeomorphism.