Week 3 (due April 20)

1. Problem 6.1 in Morita.

2. Consider the Hopf bundle $S^3 \to S^2$. This is a principal S^1 bundle (see Problem 1). S^3 has a standard metric coming from its embedding into $\mathbb{R}^4$. Using this metric, we can define a connection on the Hopf bundle by postulating that the horizontal subspace is the orthogonal complement of the vertical subspace.

(a) Compute the corresponding connection 1-form on S^3 and the corresponding curvature 2-form. Use the Euler angle parametrization of S^3 (see Week 2 assignment).

(b) Choose a trivialization of the base S^2 by two charts and write the corresponding connection 1-forms ω_α and the curvature 2-forms Ω_α. Compute the integral of the curvature 2-form over S^2 (this determines the Euler class of the Hopf bundle).