Week 1 (due Jan. 13)

(Note that you have time until next Thursday to do this homework; this is because I posted this homework one day later than usual).

1. Let X be the space consisting of two circles joined at a point ("figure eight"). Sketch the universal covering space of X.

2. (a) Prove that the cone over any topological space X is contractible.

(b) Recall that the suspension of X is a topological space ΣX defined as follows: one takes a cylinder over X and contracts both the upper and lower bases to points. Equivalently, one takes a cone over X and contracts the base of the cone to a point. Show that if X and Y are homotopically equivalent, then ΣX and ΣY are also homotopically equivalent.

3. A bouquet of two spaces X and Y with marked points is a topological space which is obtained from the disconnected sum of X and Y by identifying marked points. The bouquet is denoted $X \bigvee Y$. It has an obvious marked point. Show that $\mathbb{R}^3$ with a circle removed is homotopy equivalent to $S^2 \bigvee S^1$. It is understood here that the circle is not knotted; for example, you can remove the standard circle given by the equations $x^2 + y^2 = 1, z = 0$. (Just make it plausible: you are not obliged to write an explicit formula for the homotopy equivalence.)

4. Let G be a group, and M be a transitive G-set. Fix $m_0 \in M$ and let H be the stabilizer subgroup for m_0. (I.e. $h \in H$ iff $h(m_0) = m_0$). Let $M' = G//H$; this is also a transitive G-set.

(a) Show that M and M' are isomorphic G-sets.

(b) Show that if M_1 and M_2 are isomorphic transitive G-sets, then the corresponding subgroups H_1 and H_2 are conjugate in G.