Week 2 (due Jan. 19)

1. Show that there exists a continuous map from S^1 to S^1 which cannot be continued to a continuous map from S^1 to D^2.

2. (a) Classify all coverings of the form $p : T^n \mapsto T^n$, where T^n is the n-dimensional torus.

(b) We can think of T^2 as a product $S^1 \times S^1$. Then, if we regard S^1 as a unit circle in the complex plane, it is natural to parametrize T^2 by a pair of complex coordinates z_1, z_2, both with unit absolute value. Let k be an integer. Consider two covering maps $p_i : T^2 \to T^2$, $i = 1, 2$, given by

$$p_1 : z_1 \mapsto z_1^2, \quad z_2 \mapsto z_2 z_1^k,$$

and

$$p_2 : z_1 \mapsto z_2^{-2}, \quad z_2 \mapsto z_2^{1-k} z_1.$$

Are these two covering maps equivalent? Are they equivalent to the trivial covering?

3. (a) Suppose a two-dimensional compact surface is an n-fold cover of another one. What is the relation between their Euler characteristics?

(b) Which compact surfaces can cover the Klein bottle? Which compact surfaces can cover T^2?

(c) Which compact surfaces can be covered by S^2? Which compact surfaces can be covered by $\mathbb{RP}^2$?

4. (a) Compute the homotopy groups of $\mathbb{HP}^n$ up to π_4.

(b) Is $\mathbb{HP}^\infty$ an Eilenberg-MacLane space?