Week 5 (due Feb. 9)

1. (a) Compute all homology groups (with integral coefficients) of a sphere with g handles. Use the cell complex.

(b) The same, for a sphere with k cross-caps.

(c) The same as in (b), but with coefficients in $\mathbb{Z}_2$. (You may use either the cell complex or the universal coefficient formula).

(d) Compute the homology groups of $\mathbb{RP}^n$ with coefficients in $U(1)$ (for all n).

2. Show that $\mathbb{Z}_p \times \mathbb{Z}_q = 0$ if p, q are relatively prime.

3. (a) Compute $\mathrm{Tor}(\mathbb{Z}_m, \mathbb{Z}_n)$ from the definition, for all integer m, n.

(b) Compute $\mathrm{Tor}(U(1), \mathbb{Z}_m)$ for all m.