# Week 6 (due Feb. 16)

1.(ab) Compute $\mathrm{Ext}(U(1), \mathbb{Z})$, $\mathrm{Ext}(U(1), \mathbb{Z}_m)$ and $\mathrm{Ext}(\mathbb{Z}_m, U(1))$ for all $m$. (You can use either free(=projective) or divisible(=injective) resolutions).

2. (a) Let $X$ be a topological space, and suppose $X = A \bigcup B$, where $A, B$ are open sets (possibly intersecting). Express the Euler characteristic of $X$ in terms of the Euler characteristics of $A, B$, and their intersection. (Hint: use the Mayer-Vietoris exact sequence).

(b) Let $U_i, i \in I$ be a finite open cover of $X$. Express the Euler characteristic of $X$ in terms of the Euler characteristic of $U_i$ and their intersections.

(c) Let $p : E \to B$ be a (locally trivial) fiber bundle with fiber $F$. Assume that both $F$ and $E$ are compact. Prove that $\chi(E) = \chi(B) \times \chi(F)$.

3. Determine the ring structure on the cohomology (with integral coefficients) of a sphere with $g$ handles.

4. Let $M$ be a compact orientable manifold of dimension $4k$, $k \in \mathbb{Z}$. The intersection form on the middle-dimensional cohomology $H^{2k}(M, \mathbb{R})$ is a symmetric bilinear form whose signature is called the signature of $M$ (notation: $\sigma(M)$). Compute $\sigma(T^{4k})$.