Week 8 (due March 2)

0. Note that problem 7.5 has been misstated. If you have not discovered the correct formulation yourself, please look at the new version of the problem set 7 on my home page and solve problem 7.5.

1. Let ∇ be a covariant differential for E, and let Ω be the corresponding curvature 2-form. Show that for any section s of E one has

$$\nabla(\nabla(s)) = \Omega s,$$

while for any section r of $\mathrm{End}(E)$ one has

$$\nabla(\nabla(r)) = \Omega\, r - r\, \Omega.$$

2. (a) Show that the 4-form $\mathrm{Tr}\Omega \wedge \Omega$ is closed for any connection ω and the corresponding curvature 2-form Ω.

(b) Show that locally (i.e. over a chart where the bundle E can be trivialized) this 4-form is exact.