

Week 3 (due April 20)

1. A model with a moduli space of supersymmetric vacua is described at low energies by a supersymmetric version of the sigma-model, whose target is the space of vacua. That is, the low-energy effective action has the form

$$S = -\frac{1}{2} \int d^4x g_{ij}(\phi) \partial_\mu \phi^i(x) \partial^\mu \phi^j(x) + \dots,$$

where dots denote terms containing fermions, and $g_{ij}(\phi)$ is a Riemannian metric on the target space with coordinates ϕ^i .

(a) Consider $N = 1$ SUSY gauge theory with gauge group $U(1)$ and two chiral superfields of charge $+1$. Determine g_{ij} as a function of the FI parameter. (The target space here is S^2 if $\zeta > 0$, a single point if $\zeta = 0$, and empty if $\zeta < 0$.)

(b) The same, but for two chiral superfields of charges Q_1 and Q_2 . Consider separately the cases when Q_1 and Q_2 have the same sign and when they have opposite signs.