Week 4 (due Feb. 3)

1. Let $F_A = dA + A^2$ be the field strength of the gauge field $A = A_\mu dx^\mu$ in $2n$ dimensions. Consider a closed $2n$-form

$$\Omega_{2n}(A) = \mathrm{Tr} F_A^n.$$

(a) Show that the $2n-1$-form

$$K_{2n-1}(A) = \int_0^1 dt \mathrm{Tr}(A F_{tA}^{n-1})$$

satisfies $dK_{2n-1}(A) = \Omega_{2n}(A)$. Compute K_1, K_3 and K_5.

(b) Show that the $2n-2$ form

$$G_{2n-2}(\alpha, A) = n(n-1) \int_0^1 dt (1-t) \mathrm{Tr}(\alpha d(A F_{tA}^{n-2}))$$

satisfies $\delta_\alpha K_{2n-1}(A) = dG_{2n-2}(\alpha, A)$, where δ_α is an infinitesimal gauge variation with the gauge parameter α. Compute G_0, G_2 and G_4.

2. Consider a massive Dirac spinor field in $2n-1$ dimensions in half-space. Show that depending on the sign of the mass, and using the boundary condition $(1 + \gamma_{2n-1})\psi = 0$, there may or may not be a normalizable mode bound to the boundary.