Week 6 (due Feb. 17)

Reading: Rubakov, chapter 13.

1. The instanton solution of the $SU(2)$ Yang-Mills theory can be brought to the gauge $A_0 = 0$ using a gauge transformation

$$A_\mu \mapsto \Omega A_\mu \Omega^{-1} + \Omega \partial_\mu \Omega^{-1}.$$

Let us denote $\tau = x^0$. The residual gauge freedom can be fixed by requiring $\Omega(\tau = -\infty) = 1$.

(1) Find $\Omega(\tau, \mathbf{x})$ for all τ and $\mathbf{x}$. Show that in the gauge $A_0 = 0$ we have

$$\lim_{\tau \to +\infty} A_i = \Omega_+ \partial_i \Omega_+^{-1},$$

where $\Omega_+(\mathbf{x}) = \boldsymbol{\Omega}(\tau = +\infty, \mathbf{x})$.

(2) Find $\Omega_+(\mathbf{x}) = \boldsymbol{\Omega}(\tau = +\infty, \mathbf{x})$ and evaluate

$$n_+ = \frac{1}{24\pi^2} \int d^3x \; \epsilon^{ijk} \mathrm{Tr}(\Omega_+ \partial_i \Omega_+^{-1} \; \Omega_+ \partial_j \Omega_+^{-1} \; \Omega_+ \partial_k \Omega_+^{-1}).$$

You should get $n_+ = 1$.