Week 1 (due April 14)

Reading: Becker-Becker-Schwarz 4.1-4.6.

1. (a) Consider the following generalization of the superspace action (4.29):

$$S = \frac{i}{4\pi} \int d^2\sigma d^2\theta \bar{D} Y^\mu D Y^\nu g_{\mu\nu}(Y),$$

where $g_{\mu\nu}(Y)$ is an arbitrary metric tensor on the target space. This theory is called $d = 2$ $N = (1,1)$ supersymmetric nonlinear sigma-model. Expand the fields in terms of component fields, as in (4.33) and (4.34), express the action in terms of component fields, integrate out the auxiliary fields, and obtain the action in terms of fields X^μ and $\psi^\mu_\pm$. Don't forget to expand $g_{\mu\nu}(Y)$ in powers of $\theta_\pm$! Write the answer in a covariant form, i.e. using covariant derivatives with respect to the metric $g_{\mu\nu}$.

(b) The same, but with $g_{\mu\nu}(Y)$ replaced with $g_{\mu\nu}(Y) + B_{\mu\nu}(Y)$, where $g_{\mu\nu}(Y)$ is symmetric, as before, while $B_{\mu\nu}(Y)$ is anti-symmetric.

(c) Show that the action in part (b) is invariant (up to total derivatives) under the transformation $B_{\mu\nu} \mapsto B_{\mu\nu} + \partial_\mu \lambda_\nu - \partial_\nu \lambda_\mu$, where $\lambda_\mu(Y)$ is an arbitrary 1-form on the target space. This can be done either directly in superspace, or after integrating out the auxiliary fields.