Week 4 (May 5)

1. (a) Consider the open string action (in flat coordinates) on a half-space:

$$S = \frac{1}{2} \int d\tau d\sigma g_{\mu\nu}(X)(\partial_\tau X^\mu \partial_\tau X^\nu - \partial_\sigma X^\mu \partial_\sigma X^\nu) + \int_{\sigma=0} d\tau A_\mu(X) \partial_\tau X^\mu.$$

Here $\sigma \in [0, \infty]$, $\tau \in [-\infty, \infty]$, $g_{\mu\nu}(X)$ is an arbitrary metric on target space, and $A_\mu(X)$ is an arbitrary 1-form on the target space. Suppose X^μ on the boundary $\sigma = 0$ is unconstrained. By varying the action and requiring the boundary terms in the variation to vanish, determine the boundary condition on the derivatives of X.

(b) Now consider the $N = 1$ supersymmetric version of the string, and for simplicity assume that $g_{\mu\nu}$ is constant. The bosonic part of the action is unchanged, so one finds the same boundary condition on X as before. By requiring the left-moving and right-moving superconformal currents to agree on the boundary, deduce the boundary condition for ψ_+^μ and ψ_-^μ. (You should get a total of d constraints, where d is the dimension of the target space).