Week 5 (May 12)

Reading: Polchinski, chapter 13.

1. Consider bosonic string with a target space $\mathbb{R}^{26-d} \times T^d$, where T^d is a d-dimensional torus. In suitable coordinates, T^d can be parameterized by d coordinates X^d with the identification $X^i \sim X^i + 2\pi$, $i = 1, \ldots, d$. We assume that the metric on T^d is flat and given by a constant symmetric matrix G_{ij}. Suppose also there is a B-field on T^d given by a constant anti-symmetric matrix B_{ij}. We also assume for simplicity that the metric on $\mathbb{R}^{26-d} \times T^d$ is a product metric, and that the B-field is nonzero only for components along T^d. Apply T-duality to T^d and determine G and B n the T-dual description.

2. Consider unoriented superstring in 10 dimensions compactified on a torus T^d. It can be obtained from Type IIB superstring by gauging the worldsheet parity transformation Ω. T-duality on T^d maps Ω to another symmetry transformation which we will denote Ω_d. Write down how Ω_d acts on the fields X^μ and $\psi^\mu_\pm$. Show that this action has 2^d fixed planes on the target space $\mathbb{R}^{10-d} \times \hat{T}^d$, where $\hat{T}^d$ is the dual torus. Now take the size of $\hat{T}^d$ to infinity, while staying close to one of the fixed planes. The resulting background is called an orientifold plane (of dimension $10 - d$). Write down the field identifications for such a plane.

3. Consider Type IIB superstring compactified on a torus T^d. In the resulting theory, there are states represented by strings wrapping non-contractible cycles of T^d as well as states represented by D-strings (i.e. D1 branes) wrapping the same cycles. It is clear that the net numbers of wrapped strings and D-strings are separately conserved, and from the viewpoint of the effective theory in $10-d$ dimensions there should be conserved currents corresponding to these conserved numbers. What are these conserved currents? Similarly, one can wrap any Dp-brane, $p \leq d$, on a p-cycle of T^d (either in Type IIA or Type IIB string theory) and get a conserved quantity. What is the corresponding conserved current?