Week 7 (May 26)

Reading: Polchinski, chapter 14; BBS, chapter 8.

1. (a) Consider a massless free field theory in $d = 6$ described by a closed real 3-form H with an equation of motion $H = *H$, where $*$ is the Hodge star. Determine the number of polarization states for the corresponding particle and their transformation properties with respect to the little group $SO(4)$. Show that it is five less than the number of polarization states in a pair of chiral massless spinors. Thus a theory of a self-dual 3-form, 5 scalars, and two chiral spinors has a chance to be supersymmetric (in fact, it has $N = (2, 0)$ supersymmetry).

(b) Show that the theory of a self-dual 3-form in $d = 6$ reduces, upon compactification on a circle of radius R, to a free $U(1)$ gauge theory with the standard Maxwell action. Determine how the gauge coupling depends on R.

2. The basic reference for this exercise is a paper by E. Witten, arXiv:hep-th/9503124, where M-theory was introduced. See especially section 2.

(a) Consider M-theory compactified on a square torus with sides $2\pi R_1$ and $2\pi R_2$. Upon reduction on the first circle, one gets Type IIA string theory on $\mathbb{R}^{1,8} \times S^1$ with some radius of S^1 and vanishing C_1 (the RR 1-form gauge field). The precise radius depends on whether one works in the Einstein frame or the string frame and can be deduced from the above reference. Let us denote it R_A. By performing T-duality on S^1, one gets Type IIB string theory on a circle of radius $R_B = \alpha'/R_A$ and vanishing C_0 (the RR axion). One can also exchange the circles R_1 and R_2 and perform the same manipulations. Show that either way one gets Type IIB with the same R_B, but the coupling constants are R_1/R_2 in the first case and R_2/R_1 in the second case. Show that R_B depends only on the product $R_1 R_2$ (i.e. the area of the compactification torus) and diverges when the area goes to zero. This proves S-duality of Type IIB string theory in a special case of vanishing axion.(N.B. To get this exercise right, it is important to take into account that the string coupling transforms nontrivially under T-duality, something we have not discussed in class. The transformation law can be found in a paper by T. Buscher, Phys. Lett. B 194 (1987), p. 59-62, or else in chapter 6 of Becker, Becker, Schwarz.)

(b) More generally, we may consider compactifying M-theory on a flat torus which is a quotient of $\mathbb{R}^2 \simeq \mathbb{C}$ by the lattice generated by vectors $e_1 = 2\pi R_1$ and $e_2 = 2\pi R_1 \tau$, where $\tau \in \mathbb{C}$ can be assumed to lie in the upper half-plane. Show that reducing on one circle to get Type IIA, and Tdualizing the second circle to get Type IIA leads to a Type IIB background with a nonzero axion C_0. Show that depending on which circle is taken to be the M-theory circle, one either gets $C_0 + i/g_{st}$ is equal to τ or $1/\tau$. This implies S-duality of Type IIB in general.