Homework 3 (due Oct. 26!)

In this HW we ignore anti-holomorphic variables, or equivalently work with chiral fields.

1. A chiral field in a CFT has an expansion

$$\phi(z) = \sum_{n\in\mathbb{Z}} \phi_n z^{-n-h}$$

such that $\phi_n|0\rangle = 0$ for $n > -h$. By state-operator correspondence, the vector $|\phi\rangle = \phi_{-h}|0\rangle$ completely determines the field $\phi(z)$. We also assume that $[L_{-1}, \phi(z)] = \partial\phi(z)$.

(a) (5 pts) Show that if $\phi(z)$ is a primary, then $L_n|\phi\rangle = 0$ for all $n > 0$.

(b) (15 pts) Show that if $L_n|\phi\rangle = 0$ for all $n > 0$, and $L_0|\phi\rangle = h|\phi\rangle$, then $\phi(z)$ is a primary of conformal dimension h.