Homework 7

1. (10 pts) Analyze the theory of a p-form field C_p in n dimensions, with a gauge-invariance $C_p \mapsto C + d\lambda_{p-1}$, and an action

$$S = \int d^n x \; |dC|^2.$$

Show that the theory describes a massless particle whose polarization state can be encoded in a completely anti-symmetric tensor of rank p in a Euclidean space with $n-2$ dimensions.

2. (10 pts) One cannot write down an action for a chiral scalar in 2d. But one can nevertheless define the corresponding quantum theory by specifying the Hamiltonian and equal-time commutation relations for a scalar field $\phi(x)$, where $x \in \mathbb{R}$ is the spatial coordinate. In fact, it is better to work with the field $p(x) = \partial_x \phi$. This can be justified by saying that shifting $\phi(x)$ by a constant. $\phi(x) \mapsto \phi(x) + c$, is a gauge symmetry. Write down a Hamiltonian and the commutator for $p(x)$ so that the corresponding Heisenberg equation of motion is $\partial_0 p = \partial_x p$.

3. (10pts) Consider a theory of a 4-form field C_4 in $9+1d$ and an equation of motion

$$dC_4 = \star dC_4,$$

where $\star$ is the Hodge star operator. These equations of motion have a gauge-invariance $C_4 \mapsto C_4 + d\lambda_3$ and are Lorenz-invariant. Nevertheless, there is no Lorenz-invariant and gauge-invariant action for C_4 which would give rise to such equations of motion. Instead, let B_3 be a 4-form on the 9-dimensional space obtained by restricting C_4 to a time slice. Let $H_5 = dB_4$, where the exterior derivative is understood in the 9d sense. Write down a Hamiltonian and a commutator for the field H_4 so that the corresponding Heisenberg equation of motion is essentially equivalent to above equation of motion for C_4. How many polarization states does the corresponding massless particle have?