Homework 4

1. (10 pts) Let $B$ be a constant 2-form on a $d$-dimensional torus $T^d$, and $G$ be a constant unit metric on $T^d$. Assume that the torus is given as $\mathbb{R}^d/(2\pi\mathbb{Z})^d$, so that shifting $B \rightarrow B + \alpha' N$, where $N$ is a skew-symmetric integer-valued matrix, is a symmetry. As explained in class (see also Polchinski section 8.4), the data $(G, B)$ define an even self-dual lattice $\Gamma \subset \mathbb{R}^{d,d}$, and $N$ defines an automorphism of this lattice. Let us set $G = 1$ and $B = 0$. Show that in this case an automorphism of the lattice $\Gamma$ can be identified with an element of $O(d, d, \mathbb{Z})$, and determine this element for the automorphism corresponding to the skew-symmetric matrix $N$.

2. (10 pts) Problem 8.5 in Polchinski.