

Homework 7

1. (10 pts) Consider a $U(1)$ gauge theory in four Euclidean dimensions (with the space-time assumed to be \mathbb{R}^4) and an action

$$S = \frac{1}{4g^2} \int F_{\mu\nu}^2 d^4x + \frac{i\theta}{8\pi^2} \int F \wedge F,$$

where $F = dA$ and $A = A_\mu dx^\mu$. Let

$$\tau = \frac{\theta}{2\pi} + \frac{2\pi i}{g^2}.$$

This complex coupling takes values in the upper half-plane. Show that the theory with a coupling

$$\tau' = \frac{-1}{\tau}$$

is equivalent to the original one. Use the same idea as for demonstrating T-duality: regard F as a closed 2-form, and impose the closedness condition using a Lagrange multiplier field. Then integrate out F .

2. (10 pts) Problem 13.12 in Polchinski.
3. (10 pts) Problem 14.1 in Polchinski.