1 The first law of thermodynamics

The first law says that heat transferred to an object goes into increasing the internal energy and/or work that the object does. That is, if Q is the heat transferred, ΔU is the change in internal energy, and A is work done, then

$$Q = \Delta U + A.$$

For gases and liquids $A = p\Delta V$, where p is gas pressure, and $\Delta V = V_2 - V_1$ is change in volume.

If the object both receives and gives away heat, then Q is the net heat received, i.e. the heat received minus the heat given away. Similarly, A is the net work done, i.e. if at first the object does positive work (its volume increases) and then does negative work (because its volume decreases), one has to add up both contributions.

2 Computing work

Computing work is not easy, because pressure usually changes as the volume changes. For example, suppose the temperature is held constant. (This can be achieved if the gas can freely exchange heat with the surroundings. Then the temperature of the gas is basically the temperature of the surroundings.) Then the gas law says

$$p = \frac{\nu RT}{V},$$

where ν is the number of moles and R is the universal gas constant, $R = 8.3J/(mole \cdot K)$. That is, pressure is inversely proportional to the volume. The formula $A = p\Delta V$ is applicable only for a very tiny change in V, such that p hardly changes. If the change in V is large, one has to imagine representing it as a sequence of a large number of small changes in V, and compute the work for each small change in V, taking into account that after each change in V pressure p changes a little bit. This seems like a complicated problem. Luckily, there is powerful math which solves the problem. It is called calculus. I will avoid using calculus in this course, but in case you are interested, here is the answer for total work of gas in a process where the temperature does not change:

$$A = \nu RT \log \frac{V_2}{V_1} = p_1 V_1 \log \frac{V_2}{V_1}.$$

Here V_1 is the initial volume of gas, V_2 is the final volume of gas, and p_1 is the initial pressure. The logarithm here is not the decimal one, but the natural one (with base $e = 2.71821828...$). Whatever. I will try not to use this formula, so you need not try to recall what logarithms are.

3 Some useless terminology

By the way, a process in which temperature does not change is called isothermic. A process in which pressure does not change is called isobaric. Obviously, computing work of gas in an isobaric process is much easier: it is simply $A = p(V_2 - V_1)$.

4 Internal energy and specific heat of ideal gas

The internal energy of ideal gas is

$$U = c_V T.$$

The change in internal energy for ideal gas is therefore

$$\Delta U = c_V \Delta T,$$

where c_V is specific heat (at constant volume), and ΔT is change in temperature. Of course, c_V depends on the quantity of gas one is dealing with; usually one is given c_V for one mole of gas. Specific heat per mole of a gas is different for different gases, but is usually some not very large number (like 2 or 3) times R , where $R = 8.3 J/(mol \cdot K)$ is the universal gas constant. If the gas is monoatomic (i.e. each molecule of gas is a single atom), then $c_V = 1.5R$. This is the situation for so called noble gases (like helium or argon). More common are diatomic gases, whose molecules are made of two atoms. Examples are oxygen, hydrogen, and nitrogen. For diatomic gases c_V is approximately $2.5R$.

5 Problems

1. (a) One mole of diatomic gas in a container was maintained at room temperature ($293K$) and atmospheric pressure ($10^5 Pa$). What was the volume

occupied by the gas?

(b) The gas was brought into contact with a hot object with temperature $393K$. It started expanding, with pressure kept constant, until its temperature reached $393K$. During this process it was not allowed to give away heat to the surroundings. How much work did the expanding gas perform? What was its final volume?

(c) How much heat was provided to the gas during the expansion?

(d) After expansion the gas was allowed to cool down to the room temperature, with volume kept fixed. How much heat energy did it give away in this process?

(e) Compute the ratio of work done by the gas during the expansion (see part (b)) and the heat provided to the gas (see part (c)).

2. The "Stairmaster" exercise machine claims that climbing 100 floors is equivalent to burning 400 kilocalories of energy. Compute the work I do when climbing 100 floors, assuming that each floor is 3 meters high and my mass is $85kg$. Compute the ratio of work and the energy burnt (i.e. the efficiency of human body, regarded as an engine).