Dynamical compactification from higher dimensional de Sitter space

Matthew C. Johnson
Caltech

In collaboration with:
Sean Carroll
Lisa Randall

0904.3115
Landscapes and extra dimensions

- Extra dimensions = Landscapes of lower dimensional vacua.

- Eternal inflation - transitions within 4D EFT between vacua.

- Why are some dimensions small and others large?

- What about the extra dimensions?

- Do extra dimensions play a direct role in dynamics, or just provide the possibility of different 4D physics?
Dynamical Compactification

\[ S = \frac{M_D^{D-2}}{2} \int d^D x \sqrt{-\tilde{g}(D)} \left( \tilde{\mathcal{R}}(D) - 2\Lambda - \frac{1}{2q!} \tilde{F}_q^2 \right) \]

- We will find non-singular black brane solutions that interpolate across event horizons between a D dimensional de Sitter space and a D-q dimensional open FRW universe with a stabilized q-sphere.

- These solutions can be nucleated out of D-dimensional dS space, explaining how extra dimensions became compact.

- Many types of lower-dimensional vacua exist and can be populated.
Previous work


Cosmology inside a black hole

Each element of this picture can be understood from completely vanilla black holes in 4 dimensions.
Cosmology inside a black hole

\[ ds^2 = -\frac{dR^2}{\left(\frac{2M}{R} - 1\right)} + \left(\frac{2M}{R} - 1\right) dt^2 + R^2 d\Omega_2^2 \]

Near the horizon:

\[ x = \frac{t}{4M}, \quad \tau = \sqrt{16M^2 - 8MR} \]

\[ ds^2 = -d\tau^2 + \tau^2 dx^2 + 4M^2 d\Omega_2^2 \]

2D open FRW

"compactified" 2-sphere

\( \tau = 0 \) 2D "Big-bang" is non-singular - just the event horizon.
Cosmology inside a black hole

Can continue across the horizon by taking $\tau \rightarrow i\tau$, $R$ is spacelike.

$$ds^2 = -\tau^2 dx^2 + d\tau^2 + 4M^2 d\Omega^2$$

- Event horizon separates 2D big-crunch cosmology from asymptotically flat 4D space.
- Can study in more detail.....
Dimensional reduction

\[
ds^2 = -d\tau^2 + a^2(\tau) dx^2 + R^2(\tau) d\Omega_2^2
\]

Einstein’s equations

\[
R'' + \frac{R'^2}{2R} = -\frac{1}{2R} \quad a = R'
\]

- R evolves in the potential \( V_{eff} = \frac{1}{2} \log R \) \( \Rightarrow \) \( R'' + \frac{R'^2}{2R} = -\frac{dV_{eff}}{dR} \)

- Event horizon where \( a = R' = 0 \) \( \Rightarrow \) specify solution by R at the horizon
Going outside the horizon

• Continuing across the horizon:

\[ \tau \rightarrow i\tau \]
\[ V_{eff} \rightarrow -V_{eff} \]

• This method of dimensionally reducing to a “radion” R living in lower dimensions (the open FRW) can be used to classify a wide variety of solutions.
Adding matter

- Add a 2-form: charge the black hole.

\[ V_{eff} = \frac{1}{2} \log R + \frac{Q^2}{4R^2} \]

- Now, we can stabilize R: \( AdS_2 \times S^2 \) is a solution.
Adding matter

- Add a 2-form: charge the black hole.

\[ V_{eff} = \frac{1}{2} \log R + \frac{Q^2}{4R^2} \]

- Now, we can stabilize R: \( AdS_2 \times S^2 \) is a solution.
- There is a “landscape” of vacua, one for each Q.
Adding matter

- Add a 2-form: charge the black hole.

\[ V_{\text{eff}} = \frac{1}{2} \log R + \frac{Q^2}{4R^2} \]

- Now, we can stabilize R: \( AdS_2 \times S^2 \) is a solution.
- There is a “landscape” of vacua, one for each Q.
- The black hole solutions can have multiple horizons.
Adding matter

- Add a cosmological constant

\[ V_{\text{eff}} = \frac{1}{2} \log R + \frac{Q^2}{4R^2} - \frac{\Lambda}{4} R^2 \]

- There are new “compactification” solutions.
Adding matter

- Add a cosmological constant

\[ V_{eff} = \frac{1}{2} \log R + \frac{Q^2}{4R^2} - \frac{\Lambda}{4}R^2 \]

- \( Q \) is bounded.
Adding matter

- Add a cosmological constant

\[ V_{eff} = \frac{1}{2} \log R + \frac{Q^2}{4R^2} - \frac{\Lambda}{4} R^2 \]

- Can have up to three horizons: 2 BH and 1 cosmological

This region interpolates between:
Adding matter

- Add a cosmological constant
  \[ V_{\text{eff}} = \frac{1}{2} \log R + \frac{Q^2}{4R^2} - \frac{\Lambda}{4} R^2 \]

- Can have up to three horizons: 2 BH and 1 cosmological

This region interpolates between:

4D dS and 2D FRW
Adding matter

- Add a cosmological constant

\[ V_{eff} = \frac{1}{2} \log R + \frac{Q^2}{4R^2} - \frac{\Lambda}{4} R^2 \]

- Can have up to three horizons: 2 BH and 1 cosmological

This region interpolates between:

4D dS and 2D FRW
Adding matter

- Add a cosmological constant

\[ V_{\text{eff}} = \frac{1}{2} \log R + \frac{Q^2}{4R^2} - \frac{\Lambda}{4} R^2 \]

- Can have up to three horizons: 2 BH and 1 cosmological

- Charged black holes in de Sitter are “interpolating solutions.”

- The thermal properties of de Sitter space add interesting dynamics......
Black hole nucleation

- de Sitter space is semi-classically unstable to the nucleation of charged black holes.
  \[ \Gamma = A \exp \left[ -\left( S_{\text{inst}} - S_{dS} \right) \right] \]

- The 2D region inside of each black hole is spontaneously nucleated - An example of “Dynamical Compactification.”

- Globally, an infinite number of black holes are nucleated, populating all possible 2D crunching universes.

- Future infinity of the dS space is split into many disconnected regions.

What if the lower dimensional FRW was 4D and didn’t end in a crunch?
Now enters the magic of higher dimensional GR....
A very simple theory

\[
S = \frac{M_D^{D-2}}{2} \int d^D x \sqrt{-\tilde{g}(D)} \left( \tilde{R}^{(D)} - 2\Lambda - \frac{1}{2q!} \tilde{F}^2_q \right)
\]
**Dimensional reduction**

- Assume q-dimensional spherical symmetry (D=q+4):

\[
ds^2 = \exp \left[ -\sqrt{\frac{2q}{q+2}} \frac{\phi(x)}{M_4} \right] g_{\mu\nu}^{(4)}(x) dx^\mu dx^\nu + M_D^{-2} \exp \left[ \sqrt{\frac{8}{q(q+2)}} \frac{\phi(x)}{M_4} \right] d\Omega_q^2
\]

- For magnetic flux, Maxwell equations satisfied for:

\[
F_q = Q \sin^{q-1} \theta_1 \cdots \sin \theta_{q-1} d\theta_1 \cdots \wedge d\theta_q
\]

- Can integrate over the angular coordinates on the q-sphere and go to the Einstein frame of a 4-dimensional theory:

\[
S = \int d^4x \sqrt{-g} \left[ \frac{M_4^2}{2} R - \frac{1}{2} g^{\mu\nu} (\partial_\mu \phi)(\partial_\nu \phi) - V(\phi) \right]
\]

\[
M_4 \equiv M_D \sqrt{\text{Vol}(S^q)} \quad M_D R = \exp \left[ \sqrt{\frac{2}{q(q+2)}} \frac{\phi}{M_4} \right]
\]
A landscape of lower-dimensional vacua

- The potential is given by:

$$V(\phi) = \frac{M_4^4}{2\text{Vol}(S^q)} \left[ -q(q-1)\exp\left(-\sqrt{\frac{2(q+2)}{q}} \frac{\phi}{M_4}\right) + \frac{2\Lambda}{M_D^2} \exp\left(-\sqrt{\frac{2q}{(q+2)}} \frac{\phi}{M_4}\right) + \frac{Q^2}{2} \exp\left(-3\sqrt{\frac{2q}{(q+2)}} \frac{\phi}{M_4}\right) \right],$$

where $\Lambda$ is the cosmological constant, $M_4$ is the 4-dimensional Planck mass, $q$ is a dimensionless number, $\phi$ is the field, and $\text{Vol}(S^q)$ is the volume of the $S^q$ space.

In Fig. 1, the potential Eq. 14 with $\Lambda = 0$ and $\Lambda > 0$ is plotted for various $Q$. In Fig. 2, the right-hand side of this equality is plotted for various $Q$. The figure shows the effect of increasing $Q$ on the potential landscape for both $\Lambda = 0$ and $\Lambda > 0$. The curvature, cosmological constant, and flux terms are highlighted in the equation.
A landscape of lower-dimensional vacua

- Can have lower dimensional vacua with positive, negative, or zero vacuum energy - our landscape.
- Possible to have 4D vacua (if $q = D-4$) with a small vacuum energy.
- The radius of the stabilized sphere is always less than $R \sim \Lambda^{-1/2}$
- The sphere can be small, so this is a true compactification.
- If there are multiple q-forms, there can be vacua with various numbers of compact and non-compact dimensions.

$$\frac{F^2_q}{2q!} \rightarrow \sum_{i=2}^{D-2} \frac{F^2_{q_i}}{2q_i!}$$
Solutions with a dynamical radion.

\[ S = \int d^4 x \sqrt{-g} \left[ \frac{M_4^2}{2} R - \frac{1}{2} g^{\mu \nu} (\partial_\mu \phi)(\partial_\nu \phi) - V(\phi) \right] \]

- We need to begin with an ansatz for the 4 dimensional metric:

- Homogenous + isotropic = 4 dimensional FRW with scalar field.

- Spacelike \rightarrow \text{analytic continuation} \rightarrow \text{timelike}

- Negative curvature (open)

- No curvature (flat)

- Positive curvature (closed)
Solutions with a dynamical radion.

\[ ds^2 = -d\tau^2 + a(\tau)^2 \left[ d\chi^2 + S_k^2(\chi)d\Omega_2^2 \right] \]

\[ S_k^2 = \{ \chi, \sinh \chi \} \]

- Field and Friedmann equations:

\[ \ddot{\phi} + 3 \frac{\dot{a}}{a} \dot{\phi} = \mp V' \]

\[ \left( \frac{\dot{a}}{a} \right)^2 = \frac{1}{3M_4^2} \left( \frac{\dot{\phi}^2}{2} \pm V(\phi) \right) - \frac{k}{a^2} \]
Non-singular big-bang and big-crunch

• What about big-bang and big-crunch singularities (where $a=0$)?

$$\mathcal{R} = -\frac{\dot{\phi}^2}{M_4^2} + 4\frac{V(\phi)}{M_4^2}$$

• $a=0$ is a coordinate singularity if the field energy is finite. This requires

$$\dot{\phi} \rightarrow 0 \text{ as } a \rightarrow 0$$

from

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} = \mp V'$$

• Possible for the open and flat cases. Scale factor has universal behavior:

open

$$a = \tau \text{ as } \tau \rightarrow 0$$

$$\left(\frac{\dot{a}}{a}\right)^2 \rightarrow \frac{1}{a^2}$$

flat

$$a \propto e^{H\tau} \text{ as } \tau \rightarrow -\infty$$

$$\left(\frac{\dot{a}}{a}\right)^2 \rightarrow \pm\frac{V(\phi)}{3M_4^2}$$
Non-singular big-bang and big-crunch

- What about big-bang and big-crunch singularities (where $a=0$)?

$$ \mathcal{R} = -\frac{\dot{\phi}^2}{M_4^2} + 4\frac{V(\phi)}{M_4^2} $$

- $a=0$ is a coordinate singularity if the field energy is finite. This requires

$$ \dot{\phi} \to 0 \text{ as } a \to 0 \quad \text{from} \quad \ddot{\phi} + 3\frac{\ddot{a}}{a}\dot{\phi} = \mp V' $$

- Possible for the open and flat cases. Scale factor has universal behavior:

  open
  $$ a = \tau \text{ as } \tau \to 0 $$

  flat
  $$ a \propto e^{H\tau} \text{ as } \tau \to -\infty $$

de Sitter:
Classifying solutions

- Construct solutions by first specifying the radion potential (fix $\Lambda$ and $Q$)
- Choose an open or flat metric ansatz.

open FRW:
Classifying solutions

- Construct solutions by first specifying the radion potential (fix $\Lambda$ and Q)
- Choose an open or flat metric ansatz.
- Match segments of timelike and spacelike $\tau$ across non-singular $a=0$ surfaces.

open FRW:
Classifying solutions

- Construct solutions by first specifying the radion potential (fix $\Lambda$ and $Q$)
- Choose an open or flat metric ansatz.
- Match segments of timelike and spacelike $\tau$ across non-singular $a=0$ surfaces.

open FRW:
Classifying solutions

- Construct solutions by first specifying the radion potential (fix $\Lambda$ and Q).
- Choose an open or flat metric ansatz.
- Match segments of timelike and spacelike $\tau$ across non-singular $a=0$ surfaces.

open FRW:
At large $\phi$ the dominant term in the potential is

$$V \simeq M_4^2 \Lambda \exp \left( -2 \sqrt{q \frac{q}{2(2+q)} \frac{\phi}{M_4}} \right)$$

Exponential potentials admit attractor solutions.

The metric describes the approach to D-dimensional de Sitter space as the radius of the $q$-sphere goes to infinity.
Spacelike $\tau$

\[ \frac{\ddot{a}}{a} = -\frac{1}{3M_4^2} \left( \dot{\phi}^2 + |V| \right) \]

- Scale factor is bounded. Generic choices of initial conditions lead to a singularity:

\[ \ddot{\phi} + 3\frac{\dot{a}}{a} \dot{\phi} = \mp V' \]

- Need to match the period of the scale factor to the barrier crossing time:

\[ \Delta \tau_\phi \sim \frac{1}{\sqrt{|V''(\phi_{max})|}} \quad \Delta \tau_a \sim \frac{M_4}{\sqrt{|V''(\phi_{max})|}} \]
Spacelike $\tau$ ②

• For small enough $Q$, the periods can be adjusted by moving the endpoints. For each potential there can exist one set of non-singular endpoints:

• There are two non-singular $a=0$ endpoints, and so two event horizons.
Spacelike $\tau$

- The metric interpolates between the two event horizons.
Timelike $\tau$

• For a negative minimum, there is always a spacelike singularity as perturbations are re-focused.

• For a zero or positive minimum, the field settles into the vacuum. There is no singularity.
In this region there is a 4 dimensional open FRW universe that evolves at late times to de Sitter: This could be how our universe began!
Interpolating solutions: open FRW ansatz

\[ \ddot{a} = \frac{\omega^2}{a} \]

FIG. 9: Penrose diagrams for \( \Lambda > 0 \) solutions with an asymptotically \( D \) dimensional region and a timelike will again be denoted by

\[ \omega^2 \equiv \frac{2 \, V \, \phi \, \text{critical}}{M^p \, p+2} \]
Classifying solutions

Many other solutions can be generated from other choices of the metric ansatz.

0904.3115
An aside: embedding Inflation

• Add a scalar:

\[
S = \frac{M_D^{q+2}}{2} \int d^{q+4}x \sqrt{-\tilde{g}^{(q+4)}} \left( f(\psi)\tilde{R}^{(q+4)} - 2\Lambda - \frac{h(\psi)}{2q!}\tilde{F}_q^2 \right) + \int d^{q+4}x \sqrt{-\tilde{g}^{(q+4)}} \left( -M_{\psi}^{q}k(\psi)\tilde{g}^{\mu\nu}\partial_{\mu}\psi\partial_{\nu}\psi - V(\psi) \right)
\]

• The coupling to curvature and flux induces a negative mass squared for the scalar inside an event horizon:

• This can drive an epoch of inflation.
Dynamical Compactification

- Two solutions that contain a non-singular 4 dimensional region:

  - Interpolating
  - Compactification
Dynamical Compactification

- Two solutions that contain a non-singular 4 dimensional region:

  - These solutions are analogous to the charged dS black hole and compactification solution discussed earlier.

  - Empty de Sitter space is unstable to the nucleation of these objects.

- We have answered our original question:

  What if the lower dimensional FRW was 4D and didn’t end in a crunch?
Dynamical compactification

- Interpolating solution:

- Compactification solution:
Dynamical compactification: rates

\[ \Gamma = A \exp \left[ S^{(D)}_{ds} (1 - \alpha) \right] \]

- Rates are suppressed by the de Sitter action.
- The rate for the interpolating solutions is higher when it exists.
- The rate is highest for small \( Q \) = lowest vacuum energy.
Dynamical compactification: rates

\[ \frac{F_q^2}{2q!} \rightarrow \sum_{i=2}^{D-2} \frac{F_{q_i}^2}{2q_i!} \]

We can compare rates to vacua with different dimensionality.

\[ D = 8 \]

- No large disparity between different numbers of compactified dimensions.
- Unclear what to compare.....
Decompactification transitions  (Giddings, Giddings+Myers)

- The p+2 dimensional de Sitter vacua decay back to D dimensional de Sitter space by the same instanton:

\[
\Gamma = A \exp \left[ -\left( S_{inst} - S_{dS}^{(4)} \right) \right]
\]

- The rate into a vacuum is always larger than the rate out:

\[
\frac{\Gamma_{in}}{\Gamma_{out}} = \exp \left[ |S_{dS}^{(4)}| - |S_{dS}^{(D)}| \right] \quad |S_{dS}^{(4)}| > |S_{dS}^{(D)}|
\]

- Minkowski vacua are completely stable.
Global structure of the multiverse

- Future infinity is fractally distributed among vacua with different vacuum energy and numbers of non-compact dimensions.
- Transitions occur back and forth between $p+2$ and $D$ dimensions.
- Higher-dimensional eternal inflation!
- Connecting to predictions is a very difficult measure issue.
Summary

\[ S = \frac{M_D^{D-2}}{2} \int d^Dx \sqrt{-\tilde{g}(D)} \left( \tilde{R}^{(D)} - 2\Lambda - \frac{1}{2q!} F_q^2 \right) \rightarrow \text{Landscape of vacua.} \]

Solutions interpolate between D-dimensional dS and p+2 dimensional FRW across event horizons.

These solutions are nucleated from dS - Dynamical compactification.

Transitions back and forth populate the landscape of vacua.
Future directions

- Stability analysis.
  - For $D-q = 4$, the compactification solutions have instabilities when $q > 4$ (Bousso, de Wolfe, Myers).
  - The endpoint of the instability may still be a compact manifold (warped sphere according to Kinoshita and Mukohyama).
- What about the stability of the interpolating solutions?
- What about thermodynamical stability? Can universes evaporate?
Future directions

- Stability analysis.
- Inhomogeneities.
  - Inevitably “collisions” between interpolating solutions will occur.
  - Field outside of brane will cause stimulated emission of small-charge branes (similar to Schwinger pair production).
  - These are multi-centered black brane solutions.
  - This changes the geometry - what happens to the homogeneity of the 4 dimensional FRW inside the horizon?
- Are there potentially observable effects?
Future directions

- Stability analysis.
- Inhomogeneities.
- Pre-big bang effects.
  - On the other side of the non-singular big-bang surface, extra dimensions become “large”.
  - Does this lead to any interesting effects?
Future directions

- Stability analysis.
- Inhomogeneities.
- Pre-big bang effects.
- Measure issues.

  - We have a catalog of nucleation rates. They have rather simple (and suggestive) properties.
  - Is it possible to go from this to statistical predictions for various fundamental parameters?
  - Requires an understanding of the measure - similar to eternal inflation.
Future directions

- Stability analysis.
- Inhomogeneities.
- Pre-big bang effects.
- Measure issues.
- What about standard 4D eternal inflation?
  - Membrane nucleation can occur inside of the locally 4D region, leading to the standard picture of 4D eternal inflation.
  - Subtleties due to interaction of flux d.o.f. with radion.
Future directions

- Stability analysis.
- Inhomogeneities.
- Pre-big bang effects.
- Measure issues.
- What about standard 4D eternal inflation?
- Other solutions?
  - Homogenous but anisotropic metric ansatz will generate different solutions.
  - A flat metric ansatz generates non-extremal black branes.
  - What about the other Bianchi types?
  - Bent branes?
The End.

Thanks!
Membrane nucleation inside of black branes.