A Bound on gauge couplings in the presence of gravity:

examining the AMNV bound

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Building a theory.

Effective theory/theories.

Full quantum gravity and our universe.

Different pieces of the effective theories fit together (self-consistently).

But, are all of these self-consistent chunks part of the whole picture?

Do we really need all of these pieces?
Building a theory.

Effective theory/theories.

Full quantum gravity and our universe.

It is helpful to let the gross features of the full theory inform the way effective theories are put together and vice-versa!

Let’s try to eliminate some of the pieces!
Pitfalls in effective theories.

• Not every self-consistent effective theory need be a low energy limit of a real theory of quantum gravity. Some “bad” effective theories:
  • Infinite volume moduli space.
  • Arbitrarily large rank gauge groups.
  • Potentials below the Great Divide.
  • Theories not satisfying the AMNV bound..... and many more

“Bad” = Not realized in a candidate theory of quantum gravity (i.e. string theory), and/or has pathological semi-classical effects.
Why do we care?

- Constructing “informed” effective theories is not as hard as constructing full quantum gravity, but contains some of its elements.
  - We may get some clues as to what the full theory is.
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• Constructing “informed” effective theories is not as hard as constructing full quantum gravity, but contains some of its elements.
  
  • We may get some clues as to what the full theory is.

• The string theory landscape is an effective theory full of inconsistent pieces: “Swampland.”
  
  • We may be able to reduce $10^{500}$ states to something more reasonable.

• Implications for testing string theory (i.e. using statistics of the landscape, eternal inflation, etc.).
Focus for today.

Why is gravity the weakest force in our universe?

(The Akani-HamedMotlNicholsVafa bound.)

Is it always the weakest force?

(Seemingly, yes!)
The AMNV Bound

- AMNV claim that:

1) There has to be a light charged particle with

\[ m \leq e \, m_p \]

This is a bound on the gauge coupling!

\[ e \geq \frac{m}{m_p} \]
The AMNV Bound

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\[ e \geq \frac{m}{m_p} \]

2) The effective gauge theory breaks down at a scale

\[ \Lambda \leq e \, m_p \]

(2 is a consequence of 1)
The AMNV Bound

Monopole mass must be at least:

\[ M = g_{el}^2 \int_{1/\Lambda} |\vec{B}|^2 d^3x \]

\[ |\vec{B}|^2 = g_{el}^{-4} r^{-4} \]

\[ \rightarrow M_{monopole} \sim \frac{\Lambda}{g_{el}^2} \]
The AMNV Bound

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\[ \rightarrow M_{\text{monopole}} \sim \frac{\Lambda}{g_{el}^2} \]

Now, use the AMNV bound to define the cutoff

\[ g_{\text{mag}} = \frac{1}{g_{el}} \geq \frac{M_{\text{monopole}}}{m_p} \quad \rightarrow \Lambda \leq g_{el} m_p \]
The AMNV Bound

Why do we expect the AMNV bound to be true?

If it is not true, then:

- Charged black holes can have entropy larger than their Bekenstein-Hawking entropy.
- Charged black holes cannot evaporate completely, and form planck-sized remnants.
  - These “particles” violate entropy bounds.
  - Dominate the phase space of thermodynamic systems.
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A rule appears:
Charged black holes should be able to evaporate.
Gravity is the weakest force!
The Reissner-Nordstrom solution

Charged black hole from Einstein-Maxwell theory:

\[ ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_2^2 \quad \quad f(r) = 1 - \frac{2GM}{r} + \frac{Ge^2Q^2}{r^2} \]

Roots of \( f(r) \) determine location of the horizons:

\[
\begin{align*}
\frac{M}{m_p} &< eQ & \quad \frac{M}{m_p} &= eQ & \quad \frac{M}{m_p} &> eQ \\
0, \quad r = 0 & & \quad 1, & & \quad \text{or 2 roots.}
\end{align*}
\]
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Roots of \( f(r) \) determine location of the horizons:

\[ \frac{M}{m_p} < eQ \quad \text{or} \quad \frac{M}{m_p} > eQ \]

\[ \frac{M}{m_p} = eQ \]
The Reissner-Nordstrom solution

\[ M \geq e \ Q \ m_p \]

\[ r_+ = GM + \sqrt{(GM)^2 - Ge^2Q^2} \]

\[ GM \leq r_+ \leq 2GM \]
Black Hole Evaporation: Schwarzschild

Black holes radiate at a temperature: \[ T \sim \frac{m_p^2}{M} \sim \frac{1}{r_{BH}} \]

A species of mass \( m \) is not produced until: \( T \geq m \)

(easily seen from the Boltzmann factor \( e^{-m/T} \))

The black hole loses mass:

\[ M^3(t) = M^3(0) - m^4_p t \]

and increases its temperature:

\[ T \sim \frac{m_p^2}{(M^3(0) - m^4_p t)^{1/3}} \]
Black Hole Evaporation: Reissner-Nordstrom

Hot: $T \gg m$  Hawking radiation

Both particles and antiparticles are emitted, but the field outside the horizon introduces a chemical potential:

$$e^{-\frac{1}{T}\left( m \mp \frac{e^2 Q}{r_+} \right)}$$

Black hole discharges when:

$$\frac{e^2 Q}{r_+} \geq m$$

Equivalent to the requirement that

$$A^0(r_+) \geq m$$
Black Hole Evaporation: Reissner-Nordstrom

Cold: $T \ll m$ Schwinger pair production evaporates extremal black holes

Pair production due to the field around the BH (assumed constant):

$$\frac{dQ}{dt} \sim e^{-\frac{\pi m^2 r_+^2}{eQ}}$$

Black hole discharges when: $r_+^2 \leq \frac{eQ}{m^2}$

Field energy in a Compton wavelength:

$$\frac{1}{e^2} \vec{E}^2 \cdot \chi_c^3 \sim \frac{e^2 Q^2}{r_+^4} \cdot \frac{1}{m^3} \geq m$$
An unrealistic picture of the evaporation process.

Consider a theory with a light charged particle A and an uncharged particle B.

\[(M_A, e_A) = \left( 2, \frac{1}{2} \right)\]

(violates AMNV bound)

\[(M_B, e_B) = \left( 1, 0 \right)\]

\[M(t = 0) = 7\]

\[Q(t = 0) = 2\]

No assumptions about how this was made!
An unrealistic picture of the evaporation process.

Outcome #1

$M = 1$

$Q = 1$

This is a remnant.

Stuck here unless charge or energy conservation are violated!
An unrealistic picture of the evaporation process.

Outcome #2

\[ M = 1 \]
\[ Q = \frac{1}{2} \]
An unrealistic picture of the evaporation process.

Outcome #2

\[
M = \frac{1}{2}
\]

In general there are \( N = \frac{1}{e} \) species of planck-sized remnants if the AMNV bound is not satisfied!
Why remnants are bad: grand-cannonical ensemble.

Consider an ideal gas of remnants at temperature $T$. Each particle can be “labeled” by any charge from $1 < Q < 1/e$. Assume $e$ is very small.
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For one remnant: $Z_1 = \frac{1}{e} \frac{V}{\lambda} \quad \rightarrow \quad S_1 \sim -\ln(e)$

Even one remnant can violate entropy bounds (i.e. $S \leq \frac{A}{4}$)!
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Before the remnant stage, entropy bounds are violated!
Why remnants are bad: micro-cannonical ensemble.

Total mass and charge fixed. Assume ergodicity (i.e. remnants and anti-remnants can annihilate).

Gas of particles.

Gas of remnants.

Mixed gas of remnants, particles, and evaporating black holes.
Why remnants are bad: micro-cannonical ensemble.

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Gas of particles.

Gas of remnants.

Mixed gas of remnants, particles, and evaporating black holes.

Combinatorics from variety of species beats all!
Potential Loophole

We have seen that a system in thermal equilibrium is overwhelmed by remnants.

What if the system never reaches thermal equilibrium?

In this case we can apply a different argument against remnants.
Susskind’s Argument

Divergence in entropy of Rindler space (using EFT near the horizon) is tied to the renormalization of $G$. Need to use:

$$\frac{S}{A} = \frac{1}{4G_{\text{ren}}}$$

Consider a Rindler observer in a bath of remnants at equilibrium ($G$ is related to $S/A$ at equilibrium).

The number density of remnants a proper distance $r$ from the horizon:

(one remnant species) $N_i \sim e^{-2\pi r m_p}$

($e^{S_R} \sim 1/e$ remnant species) $N_R \sim e^{S_R - 2\pi r m_p}$
Susskind’s Argument

The region \( r \leq \frac{S_R}{m_p} \) fills with remnants, \( S_R \) for each.

Assuming \( S_R \) is large, the remnants pack to the planck density.

\[
\rightarrow \frac{S}{V} \sim S_R \ m_p^3 \quad \text{Integrate over} \quad r \leq \frac{S_R}{m_p}
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\rightarrow \frac{S}{V} \sim S_R m_p^3
\]

Integrate over \( r \leq \frac{S_R}{m_p} \)

\[
\rightarrow \frac{S}{A} \sim \frac{S_R^2}{G} = \frac{1}{4G_{\text{ren}}} \quad S_R \to \infty, \quad G_{\text{ren}} \to 0
\]

We are taken to a theory without gravity!
Deriving the AMNV bound.

We now derive the AMNV bound by requiring that all black holes are kinematically able to evaporate:

\[ M_{\text{discharge}} \geq Q \cdot m_{\text{light}} \quad \Longleftrightarrow \quad e \geq \frac{m_{\text{light}}}{m_p} \]

For extremal black holes in 4-D we recover the bound:

\[ M_{\text{ext}} = m_p \, eQ \geq Q \, m_{\text{light}} \]
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What about non-extremal black holes?

Does the bound hold in arbitrary (N+1) ST dimensions?
The AMNV bound in (N+1) D

\[ S = \frac{m_{(N+1)}^{N-1}}{16\pi} \int d^{N+1}x \sqrt{g} R - \frac{1}{4e^2} \int d^{N+1}x \sqrt{g} F_{\mu\nu} F^{\mu\nu} \]

The coupling is dimensionful: \([e]_m = (3 - N)/2\)

Define a dimensionless coupling: \(\tilde{e} = e \frac{m_{(N-3)/2}}{m_{(N+1)}}\)

Conjecture that the AMNV bound becomes:

\[ \tilde{e} \geq \frac{m_{\text{light}}}{m_{(N+1)}} \]
RN in $N+1$ dimensions

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_{N-1}^2$$

$$f(r) = 1 - \frac{16\pi GM}{(N - 1)A_{N-1}r^{N-2}} + \frac{8\pi Ge^2Q^2}{A_{N-1}(N - 1)(N - 2)r^{2(N-2)}}$$

There are still just two horizons, and again we neglect the inner horizon.

$$r_{+}^{N-2} = \frac{8\pi GM}{(N - 1)A_{N-1}} + \sqrt{\left(\frac{8\pi GM}{(N - 1)A_{N-1}}\right)^2 - \frac{8\pi Ge^2Q^2}{A_{N-1}(N - 1)(N - 2)}}$$

$$\frac{8\pi GM}{(N - 1)A_{N-1}} \leq r_{+}^{N-2} \leq \frac{16\pi GM}{(N - 1)A_{N-1}}$$
Hot black hole in (N+1) dimensions.

The black hole will discharge when \( A^0(r_+) \geq m_{\text{light}} \)

\[
A^0(r_+) = \frac{e^2 Q}{(N - 2)r_+^{N-2}} \quad \rightarrow \quad M_{\text{discharge}} = \frac{(N - 1)A_{N-1}}{8\pi(N - 2)} \frac{\tilde{e}^2 m_{(N+1)}^2}{m_{\text{light}}}
\]

We require that when \( A^0(r_+) = m_{\text{light}} \), \( M_{\text{discharge}} \geq Q \cdot m_{\text{light}} \)

\[
\tilde{e} \geq \sqrt{\frac{8\pi(N - 2)}{A_{N-1}(N - 1)}} \frac{m_{\text{light}}}{m_{(N+1)}}
\]

We recover the AMNV bound!
Cold black hole in \((N+1)\) dimensions.

The black hole will discharge when

\[
\frac{\lambda_c^N}{e^2} \tilde{E}^2 \sim \frac{1}{e^2 m_{ligh}^N} \tilde{E}^2 \geq m_{ligh} \quad \tilde{E}^2 = \left( \frac{e^2 Q}{r_{+}^{N-1}} \right)^2
\]

We require that when the equality holds, \(M_{\text{discharge}} \geq Q \cdot m_{\text{light}}\)

This yields (after substituting \(r_+\)):

\[
f_1 (Q, m_{\text{light}}, N) \tilde{e}^{\frac{N-2}{N-1}} - f_2 (Q, m_{\text{light}}, N) \tilde{e}^{\frac{N-2}{N-1}} + f_3 (N) \tilde{e}^2 \geq 0
\]

Look for the zero as a function of \(Q\), extremize \(Q\) to get strictest bound.
Cold black hole in \((N+1)\) dimensions.

\[
\rightarrow \tilde{e} \geq \sqrt{\frac{8\pi(N - 2)}{A_{N-1}(N - 1)} \frac{m_{\text{light}}}{m_{(N+1)}}}
\]

We recover the AMNV bound!

Same pre-factor as the hot case!

So, we have shown in arbitrary ST dimensions that

\[
M_{\text{discharge}} \geq Q \cdot m_{\text{light}} \quad \longleftrightarrow \quad e \geq \frac{m_{\text{light}}}{m_{p}}
\]
String Theory Checks

- String theory is a consistent theory of quantum gravity.
- Need to look for situations with both closed and open strings to get a U(1) gauge field coupled to gravity.

Some examples are:

- Compactified Type-I.
- D0 branes in Type-IIA.
- Dp branes in Type-II with a compactified worldvolume.
- Stacks of Dp branes.
- Compactified heterotic strings.
Non-abelian gauge theories

- If free in the IR: there are massless gauge bosons, so any coupling is allowed.

- If free in the UV: There is confinement, so no global charges.

- If possible to Higgs down to a U(1), bound comes from massive gauge bosons: \( m_W \sim e \cdot v \leq e \cdot m_P \)
Conclusions

- Remnants are disastrous!

- Requiring that charged black holes evaporate implies the AMNV bound:

\[ M_{\text{discharge}} \geq Q \cdot m_{\text{light}} \quad \leftrightarrow \quad e \geq \frac{m_{\text{light}}}{m_p} \]

- This relation generalizes to higher dimensions:

\[ \tilde{e} = e \, m^{(N-3)/2} \quad \rightarrow \quad \tilde{e} \geq \sqrt{\frac{8\pi(N-2)}{A_{N-1}(N-1)}} \frac{m_{\text{light}}}{m_{(N+1)}} \]

- Look for new physics at a scale:

\[ \Lambda \leq e \, m_p \]

**Consistent theories of quantum gravity require that gravity is the weakest force in arbitrary ST dimension!**
Outlook: What other clues might we find?

• There are constraints on scalar potentials: inflatons and axions.

• One idea (not yet rigorous): Nix Eternal Inflation.

There are clues that the number of e-folds during inflation is bounded by entropy considerations.

This corresponds to a UV cutoff on the perturbations produced during inflation. Perhaps the large fluctuations postulated to drive eternal inflation are a semi-classical myth.

What happens when this theory breaks down? Effective theories breaking down often change the spacetime asymptotics, so the postulate is that you make black holes.

Should be able to constrain inflaton potentials. Note: potentially observable consequences!
Outlook: What other clues might we find?

- Another idea: study instantons.
  There are a number of possible thin-wall transitions (the L and R tunneling geometries), but no clear semi-classical way to rule any out.

  What are the “quantum gravitational” rules that one must impose? Are there pathologies?
  - Holography
  - String theory
  - Finite number of states
  - ADS/CFT

Study the “Great Divide” - (divide in the space of potentials)

Above the great divide, lowest CC minimum is a ground state.

Below the great divide, the nature of the theory of QG is unclear. Does this theory break down?
The End.