Physics 135c, String Theory

Final Problem Set

Spring 2005

Due Thursday 2 June 2005 in class

Seniors and graduate students: For extensions due to conflicts with early finals in other classes or related concerns, please email me and we'll try to work something out.

Optional reading: Next week we wrap up our discussion of the Virasoro operators and Lorentz charges, and then spend the rest of the week on applications to string thermodynamics. We might talk briefly about superstrings as well. In case you would like to follow along in the textbook, the relevant parts are Chapter 16 and Section 13.5.

Problems: Please do five of the following eight problems or more for extra credit.

1. Lorentz generators and Lorentz algebra. Zwiebach Problem 11.5.

2. Lightcone gauge commutator $[M^{-I}, M^{-J}]$ for the point particle. Zwiebach Problem 11.6.

3. Transformations generated by the lightcone gauge Lorentz generators M^{+-} and M^{-I}. Zwiebach Problem 11.7.

4. Lightcone gauge commutator $[M^{-I}, M^{-J}]$ for the open string.

Starting from Zwiebach Eqs. (12.160) and (12.161),

$$\begin{aligned} M^{-I} &= x_0^- p^I - \frac{1}{2}\big(x_0^I p^- + p^- x_0^I\big) - i\sum_{n=1}^{\infty}\frac{1}{n}\big(\alpha_{-n}^- \alpha_n^I - \alpha_{-n}^I \alpha_n^-\big) \\ &= x_0^- p^I - \frac{1}{4\alpha' p^+}\big(x_0^I (L_0^\perp + a) + (L_0^\perp + a) x_0^I\big) - \frac{i}{\sqrt{2\alpha'}p^+}\sum_{n=1}^{\infty}\frac{1}{n}\big(L_{-n}^\perp \alpha_n^I - \alpha_{-n}^I L_n^\perp\big), \end{aligned}$$

derive the commutator (12.162) (corrected to include a minus sign)

$$\begin{aligned} [M^{-I}, M^{-J}] = -\frac{1}{\alpha' {p^+}^2}\sum_{m=1}^{\infty}\big(\alpha_{-m}^I \alpha_m^J - \alpha_{-m}^J \alpha_m^I\big) \\ \times\Big\{m\Big[1 - \frac{1}{24}(D-2)\Big] + \frac{1}{m}\Big[\frac{1}{24}(D-2) + a\Big]\Big\}. \end{aligned}$$

You may assume the result

$$[\tilde{M}^{-I}, \tilde{M}^{-J}] = 0 \quad \text{for} \quad \tilde{M}^{-I} = x_0^- p^I - \frac{1}{2}\big(x_0^I p^- + p^- x_0^I\big).$$

This is equivalent to the point particle result derived in Problem 2, since the commutation relations obeyed by x_0^-, x_0^I, p^- and p^I are the same as those for a point particle.

5. Tachyon Potentials. Zwiebach Problem 12.11.

6. $L_0^\perp - \bar{L}_0^\perp$ as world-sheet momentum. Zwiebach problem 13.4

7. Unoriented closed strings. Zwiebach Problem 13.5.

8. Orientifold Op-planes. Zwiebach Problem 13.6