Physics 135c, String Theory

Problem Set 7

Spring 2005

Reading: Please read Zwiebach Sections 12.6–12.8 and 13.1–13.4.

Problems: Please do Problem 1 and then pick one of Problems 2 and 3 or do both for extra credit.

Also, due to an error in the original formulation of Problem 1 on the last problem set (closed circular string in an expanding universe), the due date for that problem was extended by one week. It is is now an extra credit problem this week as well.

1. Two point correlation function for a Klein-Gordon field.

We saw in class that the Fourier components $a_{\mathbf{p}}^\dagger$ and $a_{\mathbf{p}}$ appearing in the expansion of the Klein-Gordon field operator can be interpreted as creation and annihilation operators for single particles of momentum $p^\mu = (E_{\mathbf{p}}, \mathbf{p})$. Similarly, one can think of the Klein-Gordon field $\phi(t, \mathbf{x})$ as creating and annihilating single particles at position $\mathbf{x}$. However, these particles are not delta-function localized in position space, but rather are spread out over a Compton wavelength. One way to see this from the two-point vacuum correlation function

$$D(x-y) = \langle\Omega|\phi(x)\phi(y)|\Omega\rangle.$$

Since ϕ is Hermitian, this is the inner product between $\phi(x)|\Omega\rangle$ and $\phi(y)|\Omega\rangle$.

(a) Show that in the box normalization conventions of the textbook,

$$D(x-y) = \frac{1}{V}\sum_{\mathbf{p}} \frac{1}{2E_{\mathbf{p}}} e^{ip\cdot(x-y)}.$$

In the limit that the box becomes infinitely large, we have $\frac{1}{V}\sum_{\mathbf{p}} \to \int \frac{d^d\mathbf{p}}{(2\pi)^d}$, and this result becomes

$$D(x-y) = \int \frac{d^d\mathbf{p}}{(2\pi)^d} \frac{1}{2E_{\mathbf{p}}} e^{ip\cdot(x-y)}. \tag{1}$$

(b) For spacelike separations $(x-y)^2 = r^2 > 0$, there always exists a Lorentz frame in which $x^0 = y^0 = t$ and $|\mathbf{x}-\mathbf{y}| = r > 0$. In this case, specializing to $3+1$ dimensions, show that the integral (1) can be expressed as

$$D(x-y) = -\frac{1}{(2\pi)^2}\frac{1}{r}\frac{\partial}{\partial r}\int_0^\infty du \frac{\cos(mru)}{\sqrt{u^2+1}}. \tag{2}$$

(c) From Abramowitz and Stegun 9.6.21, the integral appearing in Eq. (2) is an integral representation of the Bessel function $K_0(x)$:

$$K_0(x) = \int_0^\infty dt \frac{\cos(xt)}{\sqrt{t^2+1}} \quad \text{for} \quad x > 0.$$

Look up the asymptotic form $K_0(x)$ for large x to show that the leading large r behavior of $D(x-y)$ is a decaying exponential e^{-mr}. This shows that the states $\phi(t,\mathbf{x})|\Omega\rangle$ and $\phi(t,\mathbf{y})|\Omega\rangle$ are correlated over a length

$$\Delta r \sim \frac{1}{m} = \frac{\hbar}{mc} \quad \text{(Compton Wavelength)}.$$

2. Field equations and particle states for the Kalb-Ramond field. Zwiebach Problem 10.6.

When we study the quantum particle spectrum of the closed (bosonic) string, we will find that the massless fields consist of the graviton $h_{\mu\nu}$, the dilaton ϕ, and the Kalb-Ramond field $B_{\mu\nu}$. Just as the action for a charged particle contains a term with A_μ integrated along the worldline, the full string action contains a term with the Kalb-Ramond field integrated over the worldsheet.

3. Massive vector field. Zwiebach Problem 10.7.

Here is some background for this problem. The Lagrangian density for a *complex* Klein-Gordon field φ with potential $V(\varphi)$ and charge q coupled to an electromagnetic field is

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - |\partial\varphi - iqA\varphi|^2 - V(\varphi).$$

Now suppose that the potential $V(\varphi)$ depends only on $|\varphi|$ and has a minimum at $|\varphi| = m/(\sqrt{2}q)$ for some m. Suppose further that the potential is either very steep, so that it effectively freezes $|\varphi|$, or else that we simply neglect the effects associated with deviations of $|\varphi|$ from $m/(\sqrt{2}q)$. Then we can write

$$\varphi = \frac{m}{\sqrt{2}q}e^{i\chi},$$

where χ is a real Klein-Gordon field. We substitute this into the above Lagrangian, and then forget about the potential $V(\varphi)$. This procedure leads to the Lagrangian that Zwiebach presents in this problem, provided that we identify Zwiebach's field ϕ with $q\chi/m$ and integrate the last term in his Lagrangian by parts.

The relevance of this problem to the real world is that a very similar Lagrangian describes the coupling of the Higgs boson to the four electroweak gauge fields. The result is that the Higgs is eaten by the $W^\pm$ and Z bosons, to give them mass. The photon remains massless.