Ph 106a

Homework Assignment No. 1
Due: Thursday, 11 October 2007

1. Extremizing a functional. A functional $T[y]$ is defined by

$$T[y] = \int_{x_i}^{x_f} F(y, y', y'', x)dx \tag{1}$$

where the prime denotes differentiation with respect to x. Find a necessary condition for $y(x)$ to extremize $T[y]$, with y and y' held fixed at the endpoints x_i and x_f.

2. Constrained maximization. Find the maximum of the function

$$f(x, y, z) = x + y + 2z \tag{2}$$

subject to the constraint

$$(x^2 + y^2 + z^2)^{\frac{1}{2}} = a. \tag{3}$$

Do this in two ways:

a) Use the constraint to eliminate one of the variables, and express f as a function of two independent variables. Then maximize f.

b) Use a Langrange multiplier λ. Verify that

$$\lambda = -\frac{df_{max}}{da}\,. \tag{4}$$

3. The catenary. A rope of length L and mass per unit length μ is suspended under the influence of gravity from the points $(-x_0,\ 0)$ and $(x_0,\ 0)$.

a) Use the calculus of variations to find a differential equation satisfied by the function $y(x)$ which gives the shape of the rope. (It is okay if one undetermined Lagrange multiplier appears in this equation.)

b) Verify that the differential equation is solved by

$$y(x) = a\left(\cosh\frac{x}{a} - \cosh\frac{x_0}{a}\right)\,, \tag{5}$$

and find a. (It is enough to write down a transcendental equation that can be solved for a.)