Ph 106a

Homework Assignment No. 2
Due: Thursday, 18 October 2007

1. Polar and spherical coordinates. A particle moves in the x-y plane under the influence of a central potential $V(r)$. The particle is being watched by a rotating observer, who wants to describe the motion in terms of rotating polar coordinates r and θ, which are related to x and y by

$$\begin{aligned} x &= r\cos(\theta + \omega t) , \\ y &= r\sin(\theta + \omega t) . \end{aligned} \tag{1}$$

a) Without using the Lagrangian, find the equations of motion for r and θ.

b) Use the Lagrangian to find the equations of motion.

c) Now consider a particle moving in three dimensions under the influence of a central potential $V(r)$. Use the Lagrangian to find the equations of motion, in terms of the spherical coordinates r, θ, ϕ defined by

$$\begin{aligned} x &= r\sin\theta\cos\phi , \\ y &= r\sin\theta\sin\phi , \\ z &= r\cos\theta . \end{aligned} \tag{2}$$

2. Oscillating disk. A solid disk with mass M and radius R (and, therefore, moment of inertia $I = \frac{1}{2}MR^2$) has a point mass m attached to its rim. The disk rolls without slipping on an inclined plane that makes the angle ϕ with respect to the horizontal.

a) Define a convenient generalized coordinate, and find the corresponding equation of motion.

b) Find the value of your generalized coordinate for which the disk is in equilibrium. What is the maximum value of ϕ for which an equilibrium exists?

c) Find the frequency of small oscillations about the equilibrium position. **Hint:** Use power series expansions to study the equations of motion when the generalized coordinate is close to the equilibrium value.

3. Falling off a wire. A solid sphere of mass M and radius R (moment of inertia $I = \frac{2}{5}MR^2$), is balanced on a horizontal wire, so that its center is directly above the wire. Suppose the ball falls off, rolling without slipping for as long as it remains in contact with the wire. Use the Lagrange multiplier method to find the angle at which the ball loses contact with the wire. **Hint:** Choose suitable generalized coordinates that take into account the constraint that the ball rolls without slipping, and impose the constraint that the ball is in contact with the wire using a Lagrange multiplier. Derive the equations of motion, and then use conservation of energy to find a condition that is satisfied when the Lagrange multiplier (and therefore also the force of constraint) vanishes.

4. First integral of the brachistochrone equation. The brachistochrone problem was solved by minimizing the functional

$$T[y] = \int_0^{x_f} (1 + y'^2)^{1/2}(-2gy)^{-1/2} dx \ . \tag{3}$$

Note that the integrand does not depend on x, and use this fact to obtain a first integral of the form

$$y' = f(y, C) \tag{4}$$

where C is a constant of integration.