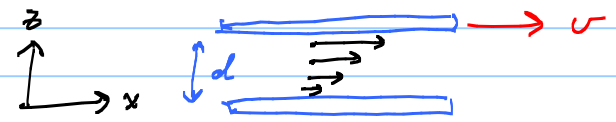
Ph 12c: Viscosity

Note Title 6/5/2012



Two plates are separated by distance d in the z direction.The top plate slides at speed v in the in the x direction, while the bottom plate stays fixed. What force is applied to the bottom plate?

$$\frac{(\text{Force})_x}{\text{Area}} = \eta\left(\frac{v}{d}\right)$$

η is the viscosity, a measure of the fluid's resistance to flow. Molasses has high viscosity; water has low viscosity.

The fluid sticks to the plates, i.e. it flows in the x direction at speed 0 for $z=0$ and speed v for $z=d$.

$\frac{v}{d} = \frac{\partial}{\partial z}(v_x)$ is the gradient in the z-direction of the x-component of velocity.

Viscosity is due to the diffusion in the z direction of the x-component of momentum, in response to a gradient of the z-momentum density:

$$\frac{(\text{Force})_x}{\text{Area}} = \begin{matrix}\text{Flux of x-momentum}\\ \text{in z-direction}\end{matrix} = -D\frac{\partial}{\partial z}\left[\frac{(\text{Momentum})_x}{\text{Volume}}\right]$$

$$= -D\frac{\partial}{\partial z}(\rho v_x) = -D\rho\frac{\partial}{\partial z}(v_x)$$

Hence $\boxed{\eta = D\rho}$. Here ρ is the mass density of the fluid, and D, the diffusion constant for x-momentum, is the same as the diffusion constant for particle concentration.

$$J = \begin{matrix}\text{particle}\\ \text{flux}\end{matrix} = -D\frac{\partial n}{\partial z} \Longleftrightarrow \begin{matrix}\text{x-momentum}\\ \text{flux}\end{matrix} = -D\frac{\partial}{\partial z}(mnv_x)$$

In order of magnitude, $D \simeq \bar{c}\, L_{MFP}$, where $\bar{c}$ is average speed and L_{MFP} is mean-free path.

Hence viscosity is $\eta = D\rho \simeq m\bar{c} L_{MFP}\, n$.

If we divide by concentration: $\frac{\eta}{n} = m\bar{c} L_{MFP} = \bar{p} L_{MFP}$.

Classically, viscosity can be arbitrarily low, but in quantum mechanics there is a lower limit.

From uncertainty principle $\frac{\eta}{n} \simeq \langle \bar{p} L_{MFP} \rangle > O(\hbar)$

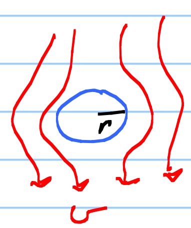
We have $\frac{\eta}{n} = O(\hbar)$ if the fluid is <u>strongly</u> coupled.

(Scattering is so frequent that momentum does not diffuse easily.) Most fluids have much larger viscosity than this lower limit. But $\eta/n = O(\hbar)$ is observed in the quark-gluon plasma created in high-energy collisions between heavy nuclei at RHIC ("relativistic heavy ion collider").

Note that for a dilute gas viscosity does not depend on concentration: $\eta \simeq n m \bar{c} L_{MFP}$ and $L_{MFP} \simeq \frac{1}{nr^2}$

$$\Rightarrow \eta \simeq \frac{m\bar{c}}{r^2} \simeq \frac{\bar{p}}{r^2}.$$

Stokes Law:



Force $F = 6\pi\eta v r$, for a ball of radius r moving a speed v through a fluid with viscosity η. The velocity gradient is $\sim v/r$ and the area is $\sim r^2 \Rightarrow$ Force $\simeq \eta v r$.

To get the 6π requires a more complicated computation. (One needs to add together the "normal stress," i.e., force perpendicular to surface, and "shear stress," i.e., force parallel to surface.)

The ratio $\eta/\rho = D$ is called "Kinematic viscosity"
(or "specific viscosity")

At $T = 300^\circ K$

$$\eta/\rho \simeq \begin{cases} 10^{-6}\ m^2 s^{-1} & \text{in water} \\ 15\times 10^{-6}\ m^2 s^{-1} & \text{in air} \\ 75\times 10^{-6}\ m^2 s^{-1} & \text{in honey.} \end{cases}$$

It is $D \simeq \bar{c}\, L_{MFP}$ where $\bar{c} \sim$ speed of sound

For H_2O $\quad \eta/n \simeq m\,\eta/\rho \simeq (30\times 10^{-27}\,kg)(10^{-6}\,m^2 s^{-1})$
$$\simeq 3\times 10^{-32}\ J\text{-}s \simeq 45(2\pi\hbar)$$
(and even larger in air).

For a process with characteristic velocity and length scale, there is a dimensionless number ("Reynolds number"):

$$Re = \frac{(\text{speed})(\text{distance})}{\eta/\rho} \sim \frac{v r}{D} \sim \frac{v r}{\bar{c}\, L_{MFP}} \sim \frac{r}{L_{MFP}}$$
(for $v \sim$ sound speed).

$$Re = \frac{(\rho r^3) v^2/r}{\eta v r} \approx \frac{\text{inertial force}}{\text{viscous force}}$$

Airplane in air: $\dfrac{(30\,m)(250\,m/s)}{15\times 10^{-6}} \sim 5\times 10^8$

Man in water: $\dfrac{(1\,m)(1\,m/s)}{10^{-6}} \sim 10^6$

Bacterium in water: $\dfrac{(10^{-6}m)(30\times 10^{-6}\,m/s)}{10^{-6}} \sim 3\times 10^{-5}$

For bacterium, viscous forces dominate: in this world, Aristotle was right... You need to push with constant force to move an object with constant speed.