Until 1995, $^4He$ was the closest thing to an ideal BEC that had been seen in the lab. But since 1995, a variety of experiments have been done that produce and study BEC in dilute atomic gases. Because the gas is dilute, atomic collisions are rare (though still not completely negligible), so that the theory of an ideal gas of bosons applies pretty accurately. These experimental developments have had a transformative effect on atomic physics.

References on the Ph12c website:

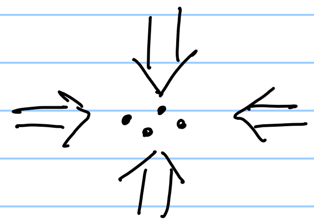
"BEC Homepage" (www.colorado.edu/physics/2000/bec)
– A website, with animations, prepared by E. Cornell and C. Wieman

"Bose-Einstein Condensation" by W. Ketterle et al., Physics World (1997)

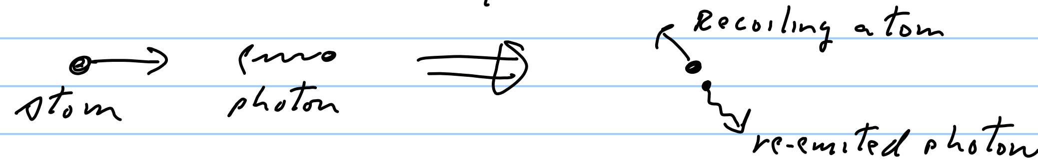
(Cornell, Wieman, and Ketterle shared the 2001 Nobel Prize for achieving BEC in atomic gases.)

The experiments described in these references are done with $^{87}Rb$ (Cornell and Wieman) and $^{23}Na$ (Ketterle). The gas is contained in a magnetic bottle (the atoms have magnetic moments, so they can be trapped using an inhomogeneous magnetic field) and has a density of about $10^{14}\ cm^{-3}$, for which the Einstein temperature is $T_E \simeq 1\ \mu K$. How can such an ultracold temperature be reached?

It is done in two steps. The first step is laser cooling with "optical molasses."

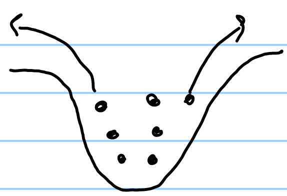


The atomic cloud is surrounded by lasers, and the light's frequency is slightly below a frequency that the atoms absorb.



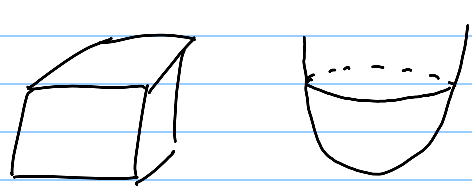

For an atom moving toward a laser, the light is slightly blue-shifted, and gets absorbed. The photon is emitted isotropically. Therefore, the collision between atom and photon is more likely to slow the atom down than speed it up. By this method, atoms can be cooled to about $100\,\mu K$.

The second step is <u>evaporative</u> cooling. The



height of the trap lowers gradually, so some atoms escape, and those with higher energy are more likely to escape than those with lower energy. Atomic collisions occur often enough to "rethermalize" the remaining atoms (if the density is high enough and the cooling is slow enough) so the atomic velocities remain thermally distributed, with a steadily declining temperature.

By this method $T$ is reduced from $T \sim 100\,\mu K$ to $T \sim T_E \sim 1\,\mu K$ (or even lower), with $N \sim 10^7$ or more atoms remaining in the BEC.



Note that the gas is confined inside a harmonic well, rather than a box with sharp walls

Therefore, we should redo our calculation of the relationship between the Einstein Temp. $\tau_E$ and the number of atoms $N$.

For an atom in an isotropic 3D trap, the Hamiltonian is

$$H = \frac{1}{2m}\left(p_x^2 + p_y^2 + p_z^2\right) + \frac{1}{2} m\omega_0^2 \left(x^2 + y^2 + z^2\right)$$

This is equivalent to three uncoupled harmonic oscillators. Orbitals are labeled by

$$n_x, n_y, n_z = 0, 1, 2, \ldots$$

(the excitation levels of the three oscillators) and the orbital's energy is

$$E(n_x, n_y, n_z) = \hbar\omega_0 (n_x + n_y + n_z)$$

(where the energy of the ground orbital is defined to be zero). In a BEC, with activity $\lambda \cong 1$, the number of atoms in excited orbitals is:

$$N_e(\tau) = \sum_{n_x, n_y, n_z = 0}^{\infty} \frac{1}{\exp[E(n_x, n_y, n_z)/\tau] - 1}$$

$$= \sum_{n_x, n_y, n_z = 0}^{\infty} \frac{1}{\exp[\hbar\omega_0 (n_x + n_y + n_z)/\tau] - 1}$$

If $\frac{\hbar\omega_0}{\tau} \ll 1$, we approximate sum by an integral

$$N_e(\tau) = \left(\frac{\tau}{\hbar\omega_0}\right)^3 I \, , \text{ where } I = \int_0^\infty dx\,dy\,dz \, \frac{1}{e^{x+y+z} - 1}$$

The Einstein Temp $\tau_E$ is such that $N_e(\tau_E) = N$, or $\tau_E = I^{-\frac{1}{3}} \hbar\omega_0 N^{\frac{1}{3}} = (0.94)\,\hbar\omega_0 N^{\frac{1}{3}}$.

Since $N_e(\tau) \propto \tau^3$, we have $N_e(\tau) = (\tau/\tau_E)^3$ for $\tau \le \tau_E$

The expression $\tau_E = (0.94)\,\hbar\omega_0 N^{1/3}$ for the Einstein temp. in a harmonic well looks different than the expression $\tau_E = \frac{2\pi\hbar^2}{m}\left(\frac{N}{(2.61)V}\right)^{2/3}$ for the Einstein temperature in a box. But in both cases the criterion for BEC is similar — namely, that the interparticle distance is comparable to the deBroglie wavelength.

Since the potential energy is $V = \frac{1}{2} m\omega_0^2 r^2$, at temp $\tau$ the typical potential energy is $V \simeq \tau \sim m\omega_0^2 r^2 \Rightarrow r \simeq \left(\frac{\tau}{m\omega_0^2}\right)^{1/2}$

and the typical distance between particles is

$R = \left(\frac{N}{V}\right)^{-\frac{1}{3}} \simeq r N^{-\frac{1}{3}} \simeq N^{-\frac{1}{3}}\left(\frac{\tau}{m\omega_0^2}\right)^{\frac{1}{2}}$. Comparing to the wavelength $\lambda_{dB} \simeq \frac{\hbar}{\sqrt{m\tau}}$, we find $R \simeq \lambda_{dB}$ for

$\tau \simeq \hbar\omega_0 N^{\frac{1}{3}}$, matching $\tau_E$ up to an $O(1)$ factor

But note that for $\tau \simeq \tau_E$, the spatial width of the ground orbital is small compared to $r$. We know from Ph 126 that the width of the oscillator's Gaussian ground state is

$$\Delta x \simeq \sqrt{\frac{\hbar}{m\omega_0}} ,$$

while at $T \simeq T_E$, the size of the ball of gas in the harmonic trap is

$$r \simeq \left(\frac{T_E}{m\omega_0^2}\right)^{\frac{1}{2}} \simeq \left(\frac{\hbar\omega_0 N^{1/3}}{m\omega_0^2}\right)^{\frac{1}{2}} = N^{1/6}\sqrt{\frac{\hbar}{m\omega_0}}$$

and hence $\quad \dfrac{(\Delta x)_{\text{ground}}}{r_{\text{thermal}}} \simeq N^{-1/6}$

when the gas cools to $T \lesssim T_E$ and the condensate appears, it is a dense "pit" inside a more diffuse "cherry."

If the trap turns off, cloud of atoms spreads, but the pit spreads more slowly — due to the quantum uncertainty in momentum rather than thermal motion. The distribution of atoms can be imaged with a laser flash, or less destructively by the dispersive method described in Ketterle's article. Thus one can "see" the multiply occupied ground state wave function in the trap (though actually interactions among the atoms modify its width, in a calculable way). The ground orbital contains about $10^7$ atoms in a spatial volume $\sim 10^{-7}\,\text{cm}^3$

Ketterle also describes the operation of an "atom laser" in which two condensates collide, producing an interference pattern. This is "real time" interference, as for classical light, in contrast to the interference pattern for a single atom moving through a double slit, where the experiment is repeated many times to build up an interference pattern.