Maxwell's Demon

Ref: C. Bennett Sci. Am. 257, 108-116 (1987)

H.S. Leff and A.F. Rex, "Maxwell's Demon: Entropy Information Computing" (Adam Hilger, Bristol, 1990)

— collection of reprints

Key steps in the history of Maxwell's demon.

1871 - Maxwell describes him in "Theory of Heat"
1929 - Szilard considers entropy associated with acquisition of information

1961 - Landauer considers thermodynamics of computation, and recognizes that erasure of information is an irreversible process

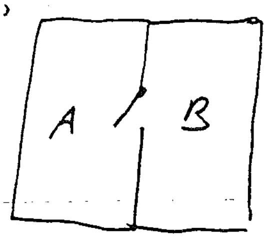
1973 - Bennett finds that computation can be performed in reversible steps

1982 - Bennett shows that, because of dissipation when demon's memory is erased, Maxwell's demon does not violate the 2nd law

the definitive account, thus far.

Maxwell - 1871

="Limitations of the 2nd Law of Thermo"



Gas in box, partitioned with a shutter separating 2 halves. Demon observes speed of molecules that approach shutter. He allows

- Fast ones pass from A to B
- Slow ones pass from B to A

thus: A cools and B heats up

This can be done while expending negligible work

Szilard - 1929

We need to ask -- Does the demon dissipate heat (generate entropy) in carrying out his demonic task?

Szilard says <u>yes</u>. He invents the idea (implicitly) that entropy is associated with <u>information</u> (made explicit by Shannon in 1949).

Szilard also invents the idea of a <u>bit</u> of information (named later, by Tukey) and says explicitly that acquiring a <u>bit</u> of information generates

$$S = k_B \ln 2$$

But Szilard's discussion is vague about the details -- Is it <u>measurement</u> that generates entropy? (This is how Szilard was usually interpreted in the next few decades) Is it remembering the information, or forgetting it, that is irreversible?

Anyway — Szilard raises the key questions:

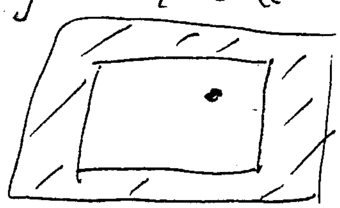
measurement
information
memory

Need to understand reversibility of these

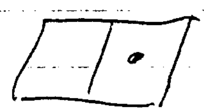
Szilard's Demon Machine

— a one-molecule gas.

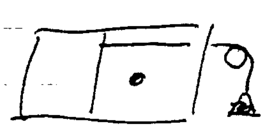
(We can only apply thermodynamics to an ensemble of many of them)



start out with one molecule in box (in thermal contact with reservoir)

1  partition box into two halves

2 Demon measures on what side of partition the gas molecule is (and records the result)

3  the partition (a movable piston) is loaded with a weight, and gas expands isothermally

Does work $W = kT \ln 2$

then repeat --

In each stage of this cycle $\Delta S = k \ln 2$ is removed from reservoir and converted to useful work $W = T \Delta S$.

There is no disposal of waste heat to reservoir at lower temperature, in violation of 2nd law (Entropy of the universe decreases)

Bennett - 1982

Bennett realizes that, to violate 2nd law, we must return demon to his initial state. This means that the demon's memory must be erased. According to Landauer (1961), erasing a bit of information is irreversible, and dissipates heat (generates $\Delta S = k \ln 2$). This heat returns to reservoir, and the entropy of the universe is unchanged

Szilard's cycle according to Bennett

- Measurement:
can be reversible (generates no entropy)
Rather, establishes correlation between demon's memory and molecule in the box.

Before measurement

Demon in standard position e.g. ↑ 0-bits

Molecule on side L or R 1-bit

After measurement

$L \uparrow$
$R \downarrow$ 1 bit

$S_{demon} = 1$ bit
$S_{mole} = 1$ bit
$S_{sys} = 1$ bit

$S_{sys} \neq S_{demon} + S_{mole}$ (because of correlation)

But establishing correlation does not change entropy (no gain or loss of information)

- Isothermal expansion

Removes $S = k_B \ln 2$ from environment (1 bit)

Increases entropy of the system by destroying the correlation

$L\uparrow \quad R\uparrow$
$L\downarrow \quad R\downarrow$ 2 bits Now $S_{sys} = S_{dem} + S_{mol}$ since uncorrelated

The entropy of universe is unchanged

- Reset the Demon

This removes 1 bit of information from the demon — so $S = k_B \ln 2$ is dissipated, and returned to the environment

Once again, the entropy of the universe is unchanged. Demon, molecule, environment are all returned to their initial state

By 1st law, dissipating heat requires that the demon do work, so we must pay back the work done in the isothermal step

— Why do we reset the demon?
  (i) simplifies the analysis — we don't have to worry about the change in the state of the demon during the complete cycle
  (ii) If we don't reset, demon acquires more and more entropy; it eventually gets disordered and cannot function.

More About Memory

Let's try to understand in more concrete (i.e. physical) terms why erasing memory is necessarily dissipative (releases heat and requires work)

Landauer (1961) introduced concept of "logical irreversibility" a logical function that has no inverse

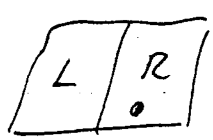
$$A \to B$$

where more than one A go to a given B — erasure is an example of logically irreversible process

(Aside: Bennett - 1973: computation <u>can</u> be reversible in principle. Get the answer, print it out (which is logically reversible) and then reverse the steps of the calculation (rather than erasing) to return computer to its initial configuration.)

How, though, are logical reversibility and thermodynamic reversibility related? We need a concrete model of a memory that records one bit

Inspired by Szilard, let memory be a partitioned box with molecule in L or R

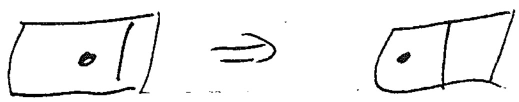


Let L be standard state — So erasure means returning molecule to L

<u>Important</u>: our erasure procedure must work the same regardless of whether state is L or R before erasure. Otherwise, we would need to record the initial state before we erase, and we would not be done until we erased the <u>record</u>, too!

An erasure procedure:

-  remove partition

-  isothermally compress

NOTE: this destroys a bit of information, and so dissipated $\Delta S = k_B \ln 2$

Indeed, $W = T\Delta S$ is done during the isothermal compression, and ΔS is released to the environment

The isothermal compression is thermodynamically reversible. But removing the partition (free expansion) is thermodynamically irreversible. So did erasure increase the entropy of the world?

Yes, in a way. Memory was initially in a definite state (either L or R) and is finally in a definite state (L). But we gave up $\Delta S = k_B \ln 2$ to the environment, associated with <u>forgetting</u> the initial state

(Bennett uses a different language — He says that memory had 2 possible states, and compressing this to 1 <u>decreases</u> entropy of the memory by same amount as increase of entropy of environment $\Rightarrow$ entropy of universe does not change.)

If we take the view that entropy of the universe increases upon erasure, then there must be a compensating decrease of entropy

this occurs when the partition is placed — Before molecule could be on either side.

After, it is definitely on one side (or the other)

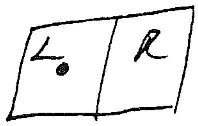
Bennett's language is appropriate if we imagine a large ensemble of Szilard's boxes, where we don't know the state of any given one — then erasure is reversible

Remark:

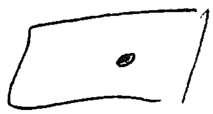
Erasure is a reversible process if and only if the information that is being erased is previously random

For our model memory — the heat $\Delta S = k_B \ln 2$ per box dissipated to the reservoir during isothermal compression exactly agrees with the $\Delta S = k_B \ln 2$ per box from counting of possible states.
If we have any information at all about the memory, the entropy destroyed during erasure is less — and there is a net increase in the entropy of the universe

Correspondingly, we can reverse the erasure process:



start with boxes in standard form L and let L isothermally expand (taking $\Delta S = k_B \ln 2$ per box from environment)



Now — place partition. Get ensemble that is $\frac{1}{2}$ L and $\frac{1}{2}$ R on the average

— Entropy of ensemble is $k_B \ln 2$ per box and information stored in the memory is <u>random</u>.