

## PHYSICS 205 Relativistic Quantum Field Theory

Prerequisites: Quantum Mechanics (e.g., Landau + Lifshitz)  
Electricity + Magnetism (e.g., Jackson)

Books (Recommended):

P. Ramond, Field Theory, Benjamin/Cummings  
C. Itzykson + J. Zuber, Quantum Field Theory,  
McGraw-Hill

(The lectures, not the books, are the basis  
of the course.)

Requirements: Problem Sets

Grading: Pass/Fail

The goal of the course is to provide an introduction to modern particle theory -- the "standard model." The course will not review the detailed experimental evidence in favor of the standard model; that is the task of Physics 231. Nor will it examine the detailed consequences and predictions of the model; for that take Physics 230 or 234. Rather, this course is intended to provide an introduction to the methodology of quantum field theory. You'll learn enough formalism to understand what the standard model is, and the technical tools needed to perform calculations.

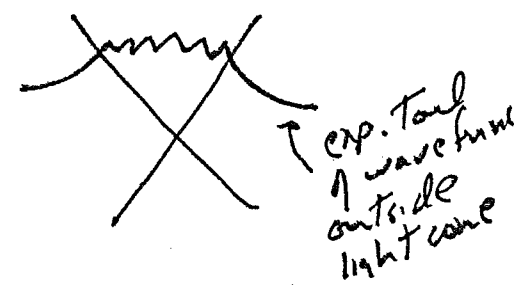
# Introduction and Motivation

Relativistic quantum field theory is the outcome of attempts to join the principles of special relativity and quantum physics; in particular, to describe the interactions of ("elementary") particles while taking relativity and quantum mechanics fully into account.

If our primary interest is really in particles, why do we bother to introduce fields? Even classically (e.g. E+M) the need for dynamical fields arises if we wish to describe the interactions of particles in a manner consistent with causality, i.e., without action at a distance.

But, in classical electromagnetism, the fundamental dynamical variables are fields and charged particles. In quantum field theory, only fields are regarded as fundamental entities, and particles arise from quantization of fields. Why this extra step? In quantum mechanics, the need for fields in describing interactions consistent with relativity is even more desperate than in classical physics. A fundamental conflict must be resolved, between locality, and the uncertainty principle.

It particles cannot be perfectly localized, what is to prevent information from leaking outside the light cone? We will see that, with fields as the dynamical variables, we can more readily construct theories that are naturally local.



Ordinarily, symmetries simplify problems (consider rotational invariance in solution to the hydrogen atom.) Why then, in relativistic quantum physics, quantum mechanics consistent with Lorentz invariance, notoriously difficult? The key difficulty is that relativistic quantum theory necessarily means field theory, and a field has an infinite number of degrees of freedom. That is really the crucial difference between particle quantum mechanics and relativistic particle mechanics, the difference between a finite and infinite number of degrees of freedom. In perturbation theory, excitations of all the degrees of freedom can occur as intermediate states. This means that (1) the perturbation expansion is complicated, and (2) there may be convergence problems in the sum over intermediate states. Infinities can occur, and the interpretation of these infinities is a subtle matter.

A final note before we begin in earnest: there is another way of constructing a consistent relativistic quantum theory, involving strings rather than (pointlike) fields. Moreover, this alternative approach seems to encompass a quantum theory of gravity (never successfully formulated in the old field theory framework), and, in fact, a unified description of all interactions. Does that mean, then, that the topic of this course is about to become obsolete, to be supplanted by string theory? Fortunately, no. It is very important to understand that a field theory can be useful, and the basis of very accurate calculations, even if it has no claim to being a fundamental description of physics at arbitrarily short distances. Old fashioned "pointlike" field

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Theory, the topic of this course, will prove useful in the future, as it has in the past, as we attempt to probe the "low-energy" consequences of the string theory (or whatever theory gives the correct description at Lplanck  $\sim 10^{-33}$  cm.)

# I. The Free Scalar Field

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Canonical Quantization	1.4
Fock Space	1.18
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Schrodinger Equation	1.25
Causality	1.28
Measurement of Quantum Fields	1.37
Fluctuations	1.43
Symmetry and Conservation Laws	1.46
Spacetime Symmetries	1.53
Internal Symmetry	1.58
Charge Conjugation	1.63
Discrete Spacetime Symmetry	1.65
Axioms of Relativistic Quantum Field Theory	1.73
The CPT Theorem	1.79

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The Self-Energy of a Static Source	2.23
The S-Matrix in a Nonlinear Field Theory	2.26
Diagrams and Symmetry Factors	2.30
The Feynman Rules	2.34
The Vacuum Energy Counterterm	2.47
Reaction Rates	2.50
Crossing Symmetry and Crossing Relations	2.61
Some Consequences of Unitarity	2.67
Two-Particle Unitarity	2.73
Dispersion Relations	2.77
Unstable Particles	2.80
Multichannel Resonance Theory	2.83
Unitarity and Statistical Mechanics	2.90A
Mass Renormalization	2.91
Field Renormalization	2.97
Coupling Renormalization	2.103
Infinite Renormalization	2.107
Higher Orders	2.113
Renormalizability	2.117
"Phenomenological" Lagrangians -- Universality	2.120
Computation of Loop Diagrams	2.126
Example: Mass renormalization	2.130
Example: Coupling Renormalization	2.133
A classification of diagrams	2.135
Green Functions and Vacuum Expectation Values	2.138
Spectral Representation of the Propagator	2.146
The Asymptotic Condition and the S-Matrix	2.150

# 3. Spin 1/2

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Properties of the Representations	3.20
The Covering Group $SL(2, \mathbb{C})$	3.25
Spin and Statistics	3.30
The Free Weyl Theory	3.40
The Free Dirac Theory	3.45
Properties of $\gamma$ Matrices	3.47
Solution to the Free Dirac Equation	3.54
Canonical Quantization of the Free Dirac Theory	3.60
Discrete Symmetries	—
Interacting Fermi Fields	—
Renormalization of Spinor Field Theory	3.100

## 4. Functional Integration

Canonical Quantization and The Path Integral	4.1
Example: The Free Scalar Field	4.14
The Semiclassical Expansion	4.23
Feynman Rules	4.28
Derivative Interactions	4.33
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The Effective Action	4.49
Ward Identities	4.55



5. Quantum Electrodynamics

Trouble with Higher Spin	5.1
Free Vector Field	5.4
Gauge Invariance	5.19
Canonical Quantization in Axial Gauge	5.25
Faddeev-Popov Ansatz	5.30
Covariant Gauges	5.35
Minimal Coupling	5.39
Geometry of Gauge Fields	5.42
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Elementary Processes	5.52
Loops in QED	5.57
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The Running Coupling Constant	5.86
Infrared Divergences	5.97

6. Spontaneous Symmetry Breakdown

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Example: Real Scalar Field	6.3
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Continuous Symmetry and the Goldstone Phenomenon	6.15
Goldstone's Theorem	6.22
Current Algebra	6.34
The Charged Weak Current	6.44
Interactions of Goldstone Bosons	6.50
The Higgs Mechanism	6.65

## 7. Nonabelian Gauge Fields

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A Model of Leptons	7.22
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How to Include Quarks	7.41
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New Physics	7.62
The Standard Model: An Appraisal	7.64

