DYON–AXION DYNAMICS

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Received 25 February 1983

We examine the coupling between magnetic monopoles and axions induced by the Witten effect, and discuss the cosmological implications of monopole–axion interactions.

The discovery by Witten [1] that a magnetic monopole of minimal magnetic charge acquires electric charge $-e\theta/2\pi$ in a θ-vacuum has led to deep insights into the fermionic structure of monopoles [2]. In this note, we point out another implication of the Witten effect – this effect induces a coupling between monopoles and axions. We compute the energy of the axion ground state in the field of a magnetic monopole and the cross section for axion–monopole scattering, in an idealized world in which there are no light electrically-charged fermions. Unfortunately, axion–monopole interactions become drastically modified if light charged fermions exist, and our calculations do not apply to this more realistic case.

But it has recently been suggested [3,4] that magnetic monopoles may carry, in addition to the usual magnetic charge, a new kind of magnetic charge, and that all particles carrying the corresponding electric charge are superheavy [4]. In this case, our calculations are relevant, and have interesting cosmological implications. In particular, we will discuss the possibility that monopoles can help the energy stored in the oscillations of the axion field [5] to relax to a cosmologically acceptable value.

To derive the connection between the Witten effect and the monopole–axion coupling, we recall that the lagrangian density of electrodynamics may contain the term

$$\mathcal{L}_\theta = (\theta e^2/4\pi^2) \boldsymbol{E}\cdot\boldsymbol{B} , \qquad (1)$$

where θ is a free parameter. In the absence of magnetic charges, this term is a total divergence, and has no physical consequences. But if a magnetic monopole is present, this term has important effects.

In unified models in which the *CP*-nonconservation of the strong interactions is suppressed by the Peccei–Quinn mechanism [6], θ becomes promoted from the status of a coupling constant to that of a field, called the axion field [7,8]. The Peccei–Quinn U(1) symmetry rotates both the QED angle θ and the strong interaction angle θ_{QCD}, but these two angles may in general differ by an amount of order one [‡1]. Nonperturbative QCD effects produce a mass m_a for the axion and fix the background value of θ_{QCD} to be close to zero. Thus, θ attains a background value θ_0 which is not

[1] Research supported in part by the US Department of Energy under Contract No. EY-76-C-02-3071.

[2] Alfred P. Sloan Fellow. Research supported in part by the National Science Foundation under Grants PHY77-22864 and PHY82-15249.

[‡1] This remark applies even to grand unified theories; for example, to an SU(5) model with Higgs 5's and 45's both Yukawa coupled to the fermions. (Our θ denotes the angle which is sometimes called $\bar{\theta}$.)

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necessarily small. For most of our discussion, we will consider the behavior of the axion field at distances from a monopole much smaller than m_a^{-1}. Therefore, we may treat the axion as massless; the only effect of QCD is to require that θ approach θ_0 at asymptotically large distances from the nearest monopole.

We can now determine how the axion field behaves near a monopole, by quantizing electromagnetism in the field of a point monopole fixed at the origin. In $A_0 = 0$ gauge, the subsidiary condition which defines gauge-invariant states is $\exp(iQ_\Lambda) = 1$, where Q_Λ is the generator of any gauge transformation such that $\exp(i\Lambda) = 1$ at the origin and at spatial infinity. If we choose $\Lambda = 0$ at $r = 0$ and $\Lambda = 2\pi$ at $r = \infty$ this condition becomes, in the (unrealistic) case in which there are no charged particles,

$$Q_\Lambda = \int d^3r \frac{\partial \mathcal{L}}{\partial \partial_0 A_i} \delta A_i$$

$$= \int d^3x [\boldsymbol{E} + (\theta e^2/4\pi^2)\boldsymbol{B}] \cdot (e^{-1} \nabla \Lambda)$$

$$= 2\pi \int_{r=\infty} d^2s \cdot [e^{-1}\boldsymbol{E} + (\theta e/4\pi^2)\boldsymbol{B}]$$

$$-\frac{1}{e} \int d^3r\, \Lambda(x) \nabla \cdot [\boldsymbol{E} + (e^2/4\pi^2)\theta \boldsymbol{B}] = 2\pi n , \quad (2)$$

when n is an integer, and the third equality has been obtained by an integration by parts. Since eq. (2) must be satisfied for any $\Lambda(x)$ consistent with the stated boundary conditions, the surface integral is required to be $2\pi n$, or

$$q/e + (\theta_0/2\pi)(2eg) = n, \quad (3)$$

where g is the magnetic charge and q is the electric charge of the monopole. Eq. (3) is Witten's condition on the electric charge of the monopole. (The Dirac quantization condition is $eg = m/2$, where m is an integer.)

From eq. (2) we may also deduce that

$$\nabla \cdot \boldsymbol{E} = -(e^2/4\pi^2) \nabla \cdot (\theta \boldsymbol{B}) , \quad (4)$$

for $r \neq 0$. We see that the axion field $\theta(r)$ determines the electric charge distribution surrounding the monopole. Oddly, the axion, a neutral scalar field, is able to screen electric charge in a background magnetic field.

The solution to eq. (4), if $\boldsymbol{B} = g\hat{r}/r^2$ is the monopole field, and there is no point electric charge at the origin, is

$$\boldsymbol{E} = (-e/8\pi^2)(2eg)\, \theta(r) \hat{r}/r^2 , \quad (5)$$

where $\theta(0) = 0$. The axion field $\theta(r)$ is strongly repelled by the monopole, because a nonzero value of θ near the origin carries a large cost in the electrostatic field energy $\frac{1}{2}E^2$. The axion ground state in the background field of a point monopole can be found by minimizing the potential energy

$$V = \int d^3r\, [\tfrac{1}{2} f_a^2 (\nabla\theta)^2 + \tfrac{1}{2} E^2]$$

$$= \int d^3r\, [\tfrac{1}{2} f_a^2 (\nabla\theta)^2 + (e^2/128\pi^4)\, \theta^2/r^4] . \quad (6)$$

In eq. (6) we have included in V the spatial gradient term, and have introduced the Peccei–Quinn symmetry-breaking scale f_a. (Note that we have chosen to normalize f_a so that $f_a\theta$ is a conventionally normalized scalar field.) We have also substituted $|eg| = \frac{1}{2}$, assuming that the monopole carries the minimal (Dirac) magnetic charge.

Now, to minimize V, we may confine our attention to spherically symmetrical functions $\theta(r)$, and make the substitution

$$z = r_0/r , \quad r_0 = e/8\pi^2 f_a , \quad (7)$$

obtaining

$$V = \frac{e f_a}{4\pi} \int_0^\infty dz\, [(d\theta/dz)^2 + \theta^2] . \quad (8)$$

The function $\theta(z)$ which approaches θ_0 as $z \to 0$ and minimizes V is clearly $\theta = \theta_0 \exp(-z)$, or

$$\theta = \theta_0 \exp(-r_0/r) , \quad r_0 = e/8\pi^2 f_a , \quad (9)$$

and the minimum value of V is ‡2

$$V_0 = (1/8\pi r_0)(e\theta_0/2\pi)^2 = e f_a \theta_0^2/4\pi . \quad (10)$$

The results (9) and (10) have been derived assuming that the monopole is pointlike, which is a reasonable approximation if the radius r_c of the monopole core is smaller than r_0. If r_c is greater than r_0, then

‡2 Periodicity in θ_0 is disguised in this expression. If we recall that eq. (3) allows θ to take the asymptotic value $\theta = \theta_0 + 2\pi n$ at $r = \infty$, for any integer n, we see that periodicity in θ_0 is restored by dyon level crossings.

eq. (9) accurately describes the axion ground state only for $r > r_c$, and eq. (10) becomes replaced by

$$V_0 \sim (1/8\pi r_c)(e\theta_0/2\pi)^2 , \quad (11)$$

the usual expression for the electrostatic self-energy of a charged sphere with $|q| = e\theta_0/2\pi$ and radius of order r_c.

Returning to the case of a point monopole, we may also calculate the cross section for axion–monopole scattering. Writing $\theta = \theta_{gs} + \theta'$ where θ_{gs} is the ground state solution in the field of a monopole at the origin, and expanding to quadratic order in θ', we obtain the effective lagrangian

$$\mathcal{L} = f_a^2 [\tfrac{1}{2}(\partial_\mu \theta')^2 - \tfrac{1}{2} r_0^2 \, \theta'^2/r^4] , \quad (12)$$

and the equation of motion

$$\ddot{\theta}' - \nabla^2\theta' + (r_0^2/r^4)\theta' = 0 . \quad (13)$$

For axion energy E satisfying $m_a \ll E \ll r_0^{-1}$, it is sensible to calculate the cross section classically, and scattering of the classical axion field by the monopole is dominated by the $l = 0$ mode. The zero-frequency $l = 0$ solution to eq. (13) which is finite at the origin is $\theta' = C \exp[-r_0/r]$, and from it we can infer that the scattering length is r_0. Hence, for $E \ll r_0^{-1}$, the classical axion–monopole scattering cross section is

$$\sigma \simeq 4\pi r_0^2 . \quad (14)$$

(In fact, this result continues to apply in the nonrelativistic limit $E < m_a$.)

The long-range tail of the axion field configuration surrounding a monopole contributes to the force between monopoles, or between monopole and antimonopole. But the leading, Coulombic, contribution is independent of r_0; it is simply the electrostatic force between electrically charged particles with $|q| = e\theta/2\pi$. The corrections are down by r_0/R, where R is the separation between the monopoles, for $R < m_a^{-1}$.

Unfortunately, much of the above analysis becomes irrelevant if light charged fermions exist. Virtual charged fermions provide another means of screening the electric charge surrounding a monopole; they allow the charge to spread out over a sphere with a radius of order m_e^{-1}, where m_e is the mass of the lightest charged fermion [2]. Thus our description of the axion ground state applies only at distances $r > m_e^{-1}$ and our analysis of axion–monopole scattering is valid only for energies $E < m_e$. The energy of the axion ground state is actually of order

$$V_0 \sim (e^2/8\pi)(\theta_0/2\pi)^2 m_e , \quad (15)$$

the electrostatic self-energy of a charged sphere with $|q| = e\theta_0/2\pi$ and radius m_e^{-1}.

Can the axion–monopole interactions described here have any detectable physical consequences? One possible application pertains to cosmology. It has recently been pointed out that in a conventional grand unified theory containing an invisible axion [8] with $f_a \sim 10^{15}$ GeV, the energy density stored in the axion field typically exceeds the critical density needed to close the universe by about three orders of magnitude [5]. An appreciable density of magnetic monopoles might encourage the background value of the axion field to relax to $\theta \sim 0$. In a suitably constructed model, θ and θ_{QCD} are nearly aligned; therefore, when the monopoles force θ toward zero, θ_{QCD} also approaches zero ‡3. By thus aiding the relaxation of θ_{QCD}, the monopoles could ameliorate the axion energy-density problem.

This effect is too small to be of interest if the coupling of the axion to the monopole is really suppressed by m_e ‡4, but could be interesting if the estimates (10) or (11) of the energy of the axion ground state apply. There is, in fact, a class of models in which (10) or (11) *do* apply.

In order to reconcile the reported observation of a monopole carrying the Dirac magnetic charge [10], $g_D = 1/2e$, with the reported observation of electric charge $q = e/3$ [11], it was recently suggested [3,4] that there is a new long-range U(1)′ gauge interaction, not yet detected, in addition to electromagnetism. According to this suggestion, the stable monopole which carries $U(1)_{em}$ magnetic charge g_D also carries a U(1)′ magnetic charge. Moreover, it has been pointed out [4] that a disastrously large cosmological abundance of stable particles carrying U(1)′ charge might be avoidable in an inflationary universe scenario [12] if all particles which carry U(1)′ charges are superheavy. In a model satisfying these criteria, there are no light

‡3 If the monopole has a color magnetic charge, there may also be a color Witten effect [9]; then monopoles drive θ_{QCD} toward zero *directly*.

‡4 At sufficiently high temperature in the early universe, weak interaction symmetries are restored, and the quarks and leptons are exactly massless; the monopole–axion coupling is thus weakened even further.

fermions to screen the U(1)′ electric charge of the monopole, and either (10) or (11) applies.

Let us now consider how a plasma of monopoles and antimonopoles would affect the axion field. If the energy of the axion ground state in the presence of a single monopole is

$$V_0 = \beta f_a \theta_0^2 , \quad (16)$$

where β is a dimensionless constant, then the energy density of the axion ground state in a plasma with monopole (and antimonopole) number density n_m is

$$U \simeq 2n_m V_0 = 2\beta(n_m/f_a)(f_a\theta_0)^2 . \quad (17)$$

This energy density has the same effect on the long-wavelength modes of the axion field as an axion mass

$$m_a^2 \simeq 4\beta(n_m/f_a) . \quad (18)$$

If the monopole–antimonopole annihilation rate is negligible, and the expansion of the universe is adiabatic, then the number density of monopoles is given by

$$n_m = \gamma T^3 , \quad (19)$$

where T is the temperature and γ is a dimensionless constant. Thus,

$$m_a^2 \simeq 4\beta\gamma T^3/f_a \quad (20)$$

is the effective axion mass.

The cosmological consequences of a temperature-dependent axion mass were analyzed in ref. [5]. The background value θ_0 of the axion field begins to oscillate coherently when $m_a \sim 3H$, where H is the Hubble parameter. For $m_a \gtrsim 3H$, the amplitude θ_0 of this oscillation decays like $m_a\theta_0^2 \propto T^3$, or, with m_a given by eq. (20),

$$\theta_0^2 \simeq (T/T_i)^{3/2}(\theta_0^2)_i . \quad (21)$$

The initial temperature T_i at which the damping of the oscillations begins is found by solving $m_a(T_i) \sim 3H(T_i)$. $H(T)$ is given in a radiation-dominated universe by

$$H \simeq T^2/Cm_p ; \quad (22)$$

here $m_p \sim 10^{19}$ GeV is the Planck mass, and $C = (0.60)N^{-1/2}$ where N is the effective number of massless spin degrees of freedom in thermal equilibrium at temperature T. Therefore we obtain

$$T_i \sim \beta\gamma(Cm_p)^2/f_a \quad (23)$$

from (20) and (22).

For a monopole of the type described above, carrying a U(1)′ magnetic charge such that the corresponding electric charge is not screened by light fermions, V_0 is given by (10) or (11). If we suppose that $r_0 > r_c$, then (10) applies, and

$$\beta = e'/4\pi , \quad (24)$$

where e' is the U(1)′ gauge coupling. If $r_0 < r_c$, then β is smaller.

The temperature T_i at which the monopole plasma begins to cause θ_0 to decay is proportional to the monopole abundance. But nonperturbative QCD effects will begin to damp θ_0 at a temperature [5]

$$(T_i)_{QCD} \sim (300\ \text{MeV})(10^{15}/f_a)^{1/6} , \quad (25)$$

and unless

$$\beta\gamma \gtrsim 10^{-22}(10^{15}\ \text{GeV}/f_a)^{5/6} \quad (26)$$

is satisfied, $(T_i)_{QCD}$ is greater than T_i given by eq. (23); hence the damping of the oscillations of the axion field is dominated by the nonperturbative QCD effects, and the role of the monopoles is insignificant. Limits on the density of monopoles today [‡5] require $\gamma \lesssim 10^{-25}$; if γ satisfies this bound, then the effect of the monopoles on the energy density stored in the axion field is negligible for $f_a \gtrsim 10^{11}$ GeV, even if β is of order 1. Because this energy density was found to be dangerously large [5] only for $f_a \gtrsim 10^{12}$ GeV, we conclude that the monopoles cannot ameliorate the axion energy-density problem.

But to reach this conclusion, we assumed that $\gamma \lesssim 10^{-25}$ was always satisfied. In a scenario in which a much larger initial density of monopoles was subsequently reduced by monopole–antimonopole annihilation, the effect of the monopoles could be more important. Suppose annihilation occurs at a temperature $T_a > (T_i)_{QCD}$. (For example, there are models [14] in which monopoles are expected to annihilate at the weak interaction scale, $T_a \sim 1$ TeV.) If $T_i > T_a$, or

$$\beta\gamma > 10^{-18}(f_a/10^{15}\ \text{GeV})(T_a/1\ \text{TeV}) , \quad (27)$$

‡5 This mass-independent limit is obtained by combining bounds based on the energetics of the galactic magnetic field with bounds based on the monopole mass density. See ref. [13].

then, prior to annihilation [‡6], the monopole plasma reduces θ_0^2 to

$$\theta_0^2 \sim (T_a/T_i)^{3/2} \sim [(10^{18}\beta\gamma)^{-1}(f_a/10^{15}\text{ GeV})(T_a/1\text{ TeV})]^{3/2} . \quad (28)$$

According to ref. [5], the energy stored in the oscillations of the axion field today will be acceptably small if

$$\theta_0^2 \lesssim 10^{-3}(10^{15}\text{ GeV}/f_a)^{7/6} \quad (29)$$

is satisfied for $T > (T_i)_{QCD}$. The monopole plasma reduces θ_0^2 to a value consistent with (29) provided that

$$\beta\gamma \gtrsim 10^{-16}(f_a/10^{15}\text{ GeV})^{16/9}(T_a/1\text{ TeV}) . \quad (30)$$

It is not unreasonable to expect a monopole abundance satisfying (30) to have been produced in a phase transition at $T \sim M_{GUT}$, where M_{GUT} is the grand unification mass scale [13]. It is somewhat more awkward, though not impossible, to devise an inflationary cosmological scenario in which the monopole abundance satisfies (30) while, at the same time, the abundance of the stable particles carrying U(1)′ electric charge is acceptably small [‡7].

If the monopoles annihilate at a temperature $T_a < (T_i)_{QCD}$, and (26) is satisfied, then an analysis similar to that in ref. [5] shows that the axion energy density today, relative to the critical density, is

$$\Omega_a = \rho_a/\rho_c \sim 10^{-19}(\beta\gamma)^{-1}(f_a/10^{15}\text{ GeV})^2 . \quad (31)$$

To derive (31) one assumes that a negligible amount of entropy is produced when the monopoles finally annihilate. Significant entropy production serves to reduce Ω_a further, but also dilutes the baryon abundance.

The above discussion requires one further qualification. We have assumed that θ and θ_{QCD} coincide, to roughly $(30)^{-1}$ accuracy; otherwise, the monopoles, by pushing θ to zero, will force θ_{QCD} to a nonzero value, and the axion energy-density problem will not be ameliorated. It is not clear how this coincidence can be achieved naturally, without some fine tuning, in a model in which all particles with U(1)′ electric charges have superheavy masses.

The interactions between monopoles and axions induced by the Witten effect appear to have interesting cosmological consequences only under rather contrived circumstances. But if the discovery of a magnetic monopole with the Dirac magnetic charge and the discovery of isolated objects with one-third-integer electric charge are both confirmed, then we will have some cause to take seriously the suggestion that cosmological monopoles hasten the relaxation of the axion field.

We thank Savas Dimopoulos, David Kaplan, and Mark Wise for useful discussions.

[‡6] Monopole–antimonopole annihilation at temperature T_a may generate a significant amount of entropy. This entropy dilutes the baryon abundance, but has no effect on the axion abundance, if T_a is greater than $(T_i)_{QCD}$.

[‡7] The monopoles must be light enough so that an appreciable monopole abundance is generated during reheating.

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