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Monopoles in 1983*

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1. Introduction

Interest in magnetic monopoles is higher in 1983 than in any previous year in recorded history. To document this claim, I have plotted in Fig. 1a interest in monopoles as a function of time over the past eleven years. "Interest" is defined as the number of high energy physics preprints written per year with the words "monopole(s)" or "dyon(s)" in the title.[1] While this accounting surely does not include all papers written about monopoles, I expect it to accurately reflect historical trends. (The 1983 total was obtained by renormalizing the number of papers written through the end of August.)

Prior to 1974, a plot of interest in monopoles against time reveals only background noise. A signal turned on in that year, which peaked in 1976, then declined slowly. The current resurgence began in 1980.

Each of the two peaks in Fig. 1a is due to several convergent factors, as is better appreciated if we divide interest into several categories. Fig. 1b is a plot of experimental interest. (This category includes all papers on the physics of monopole detection.) The Price[2] and Cabrera[3] peaks are clearly visible. Experimental interest responds quickly when there is a reason, but tends to drop to the background noise level when the reason goes away.

Theorists, on the other hand, can maintain interest without a reason. Fig. 1c is a plot of interest in the general theory of monopoles in nonabelian gauge theories (excluding the interaction of fermions and monopoles, which is treated as a separate category). The signal turned on with the pioneering papers of 't Hooft[4] and Polyakov[5], and has held steady since 1976. Of course, a continuous influx of new theoretical discoveries was needed to sustain this signal.

One sees from Fig. 1d that theoretical interest in the role of monopoles in cosmology and astrophysics turned on in 1980, and was already substantial before receiving a big boost from the Cabrera[3] event in 1982.

The most spectacular recent trend is revealed by Fig. 1e, in which interest in monopole-fermion interactions and scattering off monopoles is plotted. This subject attracted transitory interest in the late 1970's, but became a major industry only during the past year, following several seminal papers[6-9] which appeared in 1981 and early 1982.

One concludes from the above historical data that magnetic monopoles are interesting. One reason monopoles are interesting is that they can tell us something about particle physics at extremely high energies.

The existence of magnetic monopoles is a very general consequence of the (grand) unification of the fundamental interactions. Any model of particle physics in which the standard low-energy gauge group $SU(3)_{color} \times [SU(2) \times U(1)]_{electroweak}$ is embedded in a semisimple gauge group which is spontaneously broken at a large mass scale M necessarily contains magnetic monopoles. (For a review, see Ref. 10-11.) The size and mass of the monopole are determined by the symmetry breaking scale M; typical properties of a monopole in a unified model are:

Charge: $g = 1/2e$ (Dirac Charge),

Size: $r \sim (eM)^{-1}$,

Mass: $m \sim (4\pi/e)M$.

Obviously, if the magnetic monopole exists, its mass m is of great intrinsic interest; from a measurement of m we would learn, at least in order of magnitude, the fundamental symmetry breaking scale at which electrodynamics becomes truly united with the other particle interactions.

Can the monopole mass m be predicted? Yes, but only if one adopts the "desert hypothesis;" that is, assumes that no unexpected new interactions or particles appear between present-day energies (of order 100 GeV) and the unification scale M. From the desert hypothesis follows the prediction $M \sim 10^{15}$ GeV, so that $r \sim 10^{-28}$ cm and $m \sim 10^{16}$ GeV. But one should remember that the desert hypothesis could easily be wrong, even if the general idea of grand unification is correct. It is sensible to keep an open mind about the monopole mass.

The possibility that magnetic monopoles exist has excited a broad range of theoretical activity. In this summary talk, I review only some of the most recent theoretical developments concerning the implications of magnetic monopoles in cosmology, astrophysics, and particle physics, which have been discussed at this conference. Rather than attempt to give an exhaustive review, I will mention only particular topics which seem to me to be especially important or interesting.

2. Monopoles and Cosmology

The existence of magnetic monopoles is a general consequence of grand unification of the fundamental interactions. But it is one thing to say that monopoles exist, and quite another to say that we have a reasonable chance of observing one. If monopoles are extremely heavy, as expected, then any monopoles around today must have been produced in the very early universe. Thus, the monopole abundance, like the helium abundance and the baryon abundance, has become a central issue in cosmology, an issue which has exerted a healthy influence on the development of cosmology during the past few years. (See ref. 12-13 for a review.)

It is natural to ask whether the monopole abundance can be predicted. It can be, but, unfortunately, there are many predictions. Here are two predictions for the number density of monopoles $n_{monopole}$:

$$n_{\text{monopole}} \sim (m/10^{16}\,\text{GeV})n_{\text{baryon}}, \tag{2.1}$$

$$n_{\text{monopole}} \sim 0. \tag{2.2}$$

Eq. (2.1) is the prediction[14] of a standard big-band cosmology in which monopoles are copiously produced in the phase transition that takes place when the temperature is $T \sim M$. Since (2.1) implies that the mass density of the universe is dominated by monopoles by some 16 orders of magnitude, and that the universe is about 10^3 years old, this prediction is sometimes called the "monopole problem." Eq. (2.2) is the prediction[13,15] of a typical inflationary cosmology[16], in which any monopoles produced during the phase transition are subsequently "inflated away," and monopole production is negligible both during the inflationary epoch and after the universe reheats. This prediction will cause no problems until a monopole is observed. But it is apparent that cosmological considerations leave us with no definite expectation for the monopole abundance.

There have been a few recent developments concerning the cosmological monopole abundance. An old suggestion[17] for suppressing the monopole abundance within the context of the standard cosmology, is that the universe may have entered a superconducting phase as it cooled. Since a superconductor tries to expel magnetic flux, monopole-antimonopole pairs would become connected together by flux tubes in this phase, and annihilate rapidly. As the universe cooled further, it might have eventually returned to a normal nonsuperconducting phase, but only after the monopole abundance had been drastically reduced.

Previously it had been believed that the monopole abundance would be quickly reduced to a negligible value in this superconductor scenario, but E. Weinberg[19] has recently suggested that a potentially interesting number of monopoles could survive until the universe re-enters the normal phase. It is crucial to consider the correlations between monopoles and antimonopoles. These correlations are actually very strong, but Weinberg suggests that, nevertheless, some monopoles get confused; unable to decide which antimonopole to pair up with, they get left behind when the pairing occurs. It thus seems possible that the superconductor scenario predicts a detectable, or perhaps even excessively large, abundance of monopoles.

A far more appealing solution to the monopole problem is offered by cosmological inflation[16]; this solution is more appealing because inflation solves many other cosmological problems as well. In the inflationary scenario, the universe undergoes exponential expansion, driven by an effective cosmological constant, *after* the phase transition in which magnetic monopoles are produced. After many e-foldings of inflation, the monopole abundance is reduced to a negligible value. Eventually the effective cosmological constant which drove the inflation becomes rapidly thermalized, and the universe reheats. Its subsequent evolution is then described by the standard cosmological model.

Within the context of the inflationary scenario, there are at least two mechanisms for producing monopoles which ought to be considered. Naive estimates have been done of the rate of thermal production of monopoles both during[20] and after[14-15] the reheating of the universe. These estimates should probably be re-examined, and done more carefully. It is also possible for monopoles to be produced by the (Hawking) quantum fluctuations during inflation. Preliminary estimates[21] indicate that the abundance of monopoles produced by Hawking fluctuations is linked in an interesting way to the magnitude of the mass density fluctuations in the early universe. It appears that the monopole abundance due to Hawking fluctuations is guaranteed to be unobservably small if the density fluctuations are as small as required by the observed isotropy of the cosmic microwave background. But this issue also deserves a closer look.

3. Monopoles and Astrophysics

Although cosmological considerations provide us with no definite prediction for the monopole abundance, the inflationary universe scenario offers the possibility that the monopole abundance is both small enough to be acceptable and large enough to be detectable. In particular, an observable monopole abundance might have been generated during or after reheating. So theoretical cosmology should not discourage an experimenter from looking for monopoles.

But various theoretical limits on the flux of magnetic monopoles in cosmic rays can be derived by considering the effect of monopoles on astrophysical processes[22]. These limits provide valuable guidance for the prospective monopole hunter.

One stringent limit, due to Parker[23], is obtained by noting that monopoles in our galaxy will be accelerated by the galactic magnetic field, and will thus dissipate the energy stored in the field. By demanding that the field energy is not substantially depleted in the time required to regenerate the field, one obtains

$$F \lesssim 10^{-15} cm^{-2} sr^{-1} sec^{-1} , \qquad (3.1)$$

if gravitational effects on the trajectory of the monopoles are ignored. (3.1) is discouraging; it says that less than one event per year should be expected in a detector which covers a football field.

However, if the monopole mass is $m \gtrsim 10^{17}$ GeV, monopoles accelerated by the galactic magnetic field will not necessarily escape from the galaxy, and the Parker limit is less restrictive[24]; it becomes

$$F \lesssim 10^{-15} cm^{-2} sr^{-1} sec^{-1} (m/10^{17}\,\mathrm{GeV}), \quad m \gtrsim 10^{17}\,\mathrm{GeV} . \qquad (3.2)$$

For $m \gtrsim 3 \times 10^{19}$ GeV, a better constraint than the Parker limit is obtained by demanding that the mass due to monopoles in our galaxy not exceed the total mass of the galaxy.[13,24]

$$F \lesssim 3 \times 10^{-13} cm^{-2} sr^{-1} sec^{-1} \, (3 \times 10^{19} \mathrm{GeV/m}) \,.$$

(Monopoles with $m \gtrsim 10^{17}$ GeV will acquire velocities of order $10^{-3}c$ from gravitational effects.) The flux limits (3.2) and (3.3) are shown together in Fig. 2.

There are two important lessons. One is that Parker's reasoning does not exclude the possibility that magnetic monopoles make up the dark matter of galactic halos, if $m \gtrsim 3 \times 10^{19}$ GeV. The other is that the galactic monopole flux could be a few orders of magnitude larger than the oft-quoted limit (3.1), so monopole search experiments which can place bounds on the flux better than $F \lesssim 3 \times 10^{-13} cm^{-2} sr^{-1} sec^{-1}$ provide some valuable information. The above remarks seem especially noteworthy when we consider that Kaluza-Klein theories actually predict the existence of monopoles in the mass range where the flux limits are weakest. (See Section 4.)

At the last monopole conference[25], there was much discussion of the possibility of evading the Parker limit. One speculation[26], that the local flux of magnetic monopoles in the solar system greatly exceeds the ambient flux in the galaxy, seems implausible on purely kinematic grounds.[27] Another, that the observed galactic magnetic field is due to magnetic charge density fluctuations[24,28], rather than persistent currents, is more difficult to analyze, but appears to face various problems. For example, it is hard to understand how such fluctuations could have been established to begin with.

The most powerful limits on the monopole flux are obtained by considering the astrophysical consequences[29-30] of the catalysis by monopoles of nucleon decay.[8-9] In particular, monopoles incident on a neutron star or white dwarf would be captured inside the star, and their catalytic action would heat the star and raise its luminosity. Thus, from observational limits on the luminosity of such stars, we can derive limits on the product of the monopole flux F and the cross section times relative velocity $\sigma\beta$ for catalysis of nucleon decay. Very conservatively, the bound obtained in this way from neutron star luminosities, is[29-30]

$$F \lesssim 10^{-22}\, cm^{-2} sr^{-1} sec^{-1} \, (\sigma\beta / 10^{-27} cm^2)^{-1} \,, \tag{3.4}$$

about seven orders of magnitude more stringent than the Parker limit, if catalysis occurs at a strong interaction rate.

How seriously should we take this bound? In evaluating it, we must first of all decide whether it is plausible that, if magnetic monopoles do exist, they catalyze nucleon decay at a strong interaction rate, as suggested by Callan[9] and Rubakov[8]. It is certainly possible to doubt the existence of the Callan-Rubakov effect. We expect it only if the new interactions associated with the monopole core violate baryon number, and one can construct models for which this is not true.[31]

I believe, however, that it is quite natural to expect the monopoles of a unified gauge theory to exhibit the Callan-Rubakov effect, even though this prediction is not completely general. We have good reasons for suspecting that baryon number is not exactly conserved (the matter-antimatter asymmetry of the universe, the baryon-number anomaly of the standard model), and therefore there is no particular reason to expect the new physics associated with the monopole core to be baryon-number conserving. Monopole hunters should nevertheless bear in mind that the existence of monopoles which fail to catalyze nucleon decay cannot be ruled out.

If the Callan-Rubakov process does occur, the catalysis cross section cannot be calculated accurately, but it seems reasonable to estimate that it is a roughly geometrical strong interaction cross section, if the relative velocity of nucleon and monopole is of order c, as in a neutron star. Thus, the bound (3.4) is discouraging indeed. An even more stringent limit can be obtained if capture of monopoles by the main-sequence progenitor of the neutron star is taken into account. This stronger limit is more mass sensitive however. Planck-mass monopoles, for example, would rarely be captured by main sequence stars.

Once captured by a neutron star, a monopole must be accelerated to a velocity of order c to escape. Harvey[30] has helpfully suggested a possible mechanism for ejecting monopoles from neutron stars, which could considerably weaken the bound (3.4).

It is generally believed that the core of a neutron star is a type II superconductor in which Cooper pairs of protons have condensed. Because the superconducting core expels magnetic flux, monopoles entering the star will eventually come to rest at the surface of the core. Typically, many magnetic flux tubes will have been trapped in the core when it went superconducting, and a monopole floating on the surface of a core will occasionally encounter the opening of a tube, and drop in, penetrating the core.[32]

It is conceivable that, deep within the core, there is an inner core in which charged pions condense. This pion condensate would also be a type II superconductor, but its flux tubes would carry considerably higher energy per unit length than the flux tubes in the proton superconductor. Also, the flux quantum in the pion condensate would be the Dirac magnetic charge carried by the monopole, rather than half the Dirac charge as in the proton superconductor. Thus, two flux tubes in the proton superconductor would coalesce at the surface of the pion condensate, and a monopole drifting down one of them would be rapidly accelerated upon entering the pion condensate, the sizable magnetic field energy stored in the flux tube being efficiently converted to monopole kinetic energy.[30] The monopole could be accelerated to a relativistic velocity, and ejected from the star! (See Fig. 3.) Hence, if we take the fullest advantage of our ignorance concerning the interiors of neutron stars, it is possible that the very stringent bound (3.4) can be evaded.

We are left with limits based on the luminosities of white dwarfs[33]; the interiors of white dwarfs are less exotic and better understood than those of neutron stars. But these limits are rather mass-sensitive; Planck-mass monopoles, for example, would not be captured by white dwarfs.

We conclude then, that the astrophysical arguments based on monopole catalysis which have been used to limit the monopole flux are very suggestive, but not completely compelling. While it seems unlikely, I think it is possible that there is an observable flux of monopoles which catalyze nucleon decay at a strong interaction rate. Terrestrial searches for such monopoles are not purposeless.

4. Monopoles and Field theory

The theory of magnetic monopoles is a triumph of modern quantum field theory. Among the remarkable discoveries of the past ten years are the following:

- Magnetic monopoles can arise as topologically stable solitons in spontaneously broken nonabelian gauge field theories.[4-6]
- Electrically charged dyons arise as quantum mechanical excitations of these monopoles[34].

- A composite of a bosonic monopole and an elementary boson can be a fermion[35].
- Monopoles can carry a nonabelian (color) magnetic field[36].
- The color magnetic field of a monopole is screened if color electric fields are confined[37,11].
- Quark confinement in quantum chromodynamics can be regarded as a consequence of the condensation of monopoles in the vacuum state[38].
- Dyons can carry the anomalous electric charge $Q = e\vartheta/2\pi$ [39].
- Monopoles can carry fractional fermion number[40].

I will not discuss further any of these "classical" contributions to monopole theory. Instead, I confine my attention to more recent developments.

Kaluza-Klein Monopoles

More than sixty years ago, Kaluza[41] proposed a novel way of unifying gravitation with other gauge interactions. His idea is that spacetime is not really 4-dimensional, but $(4 + n)$-dimensional, that $(4 + n)$-dimensional spacetime is endowed with a metric satisfying a $(4 + n)$-dimensional generalization of Einstein's equations, and that n dimensions have become "spontaneously compactified" with radii of order the Planck length. At energies much less than the Planck mass, the microscopic compact dimensions are concealed from view, but a remnant of the underlying $(4 + n)$-dimensional theory survives at low energy; the massless fields in a theory of this type include, in addition to the 4-dimensional metric, spin-one gauge fields associated with the isometry group G of the compact n-dimensional manifold M. These gauge fields are components of the $(4 + n)$-dimensional metric which have managed to avoid acquiring masses upon compactification.

Models of particle interactions based on the concept of extra spacetime dimensional still endure today, and are known as Kaluza-Klein theories.

It is natural to expect that a Kaluza-Klein theory unifying electromagnetism with gravitation will, like other unified gauge theories, contain magnetic monopoles as static topologically stable soliton solutions to the classical field equations. This expectation has been verified, and the solution has been explicitly constructed, for the simplest Kaluza-Klein theory (that originally considered by Kaluza), in which $n = 1$, the compact manifold is the circle $M = S^1$, and the isometry group is $G = U(1)$.[42]

In general, a Kaluza-Klein soliton is a metric satisfying the $(4 + n)$-dimensional field equations which approaches the vacuum solution at spatial infinity. The behavior of the solution at spatial infinity thus defines an M bundle over S^2, the boundary of three-dimensional space. If this bundle is topologically nontrivial, the soliton will be stable. In the 5-dimensional theory, the S^1 bundles over S^2 are labeled by an integer, which, it turns out, can be identified as the magnetic charge. The Dirac quantization condition arises, as we might expect, from the requirement that a singularity of the metric be a harmless coordinate singularity.

Generally, one expects a Kaluza-Klein monopole to have a mass of order $(1/e)M_{\mathrm{Planck}}$. In the 5-dimensional theory, it has been found that[42]

$$M_{\mathrm{monopole}} = \frac{1}{4}\alpha^{-1/2} M_{\mathrm{Planck}} \sim 5 \times 10^{19}\,\mathrm{GeV}\,. \tag{4.1}$$

As remarked in Section 3, this is a quite interesting monopole mass experimentally.

The monopole of the 5-dimensional theory has some unusual properties. The 4-dimensional constant-time slices of both the monopole and antimonopole solutions have handles; therefore, a monopole-antimonopole pair has a different topology than the vacuum, and cannot annihilate classically. Quantum gravity effects, which include fluctuations in topology, would presumably allow the pair to annihilate.

Also, the interactions of monopoles with other monopoles, antimonopoles, and test particles are rather strange in the 5-dimensional Kaluza-Klein theory[42]. But these strange interactions appear to be consequences of the existence of a massless scalar particle in this theory, which would be absent in a more realistic theory.

The Kaluza-Klein monopoles appear to deserve further study. The explicit monopole solution has been constructed in only the simplest Kaluza-Klein theory. Dyonic excitations of Kaluza-Klein monopoles do not seem to arise in the usual way, because there are no charged fields excited in the monopole core. It will also be interesting to study further the interactions of fermions with Kaluza-Klein monopoles[43]. We must keep in mind that quantum gravity could exert an important effect on these interactions. Since quantum gravity (e.g., virtual black holes) is not expected to conserve baryon number, it is possible that the Callan-Rubakov effect, if it occurs at all, arises in a much different way for Kaluza-Klein monopoles than for ordinary grand-unified monopoles.

Stable Multimonopoles

Until recently, static multimonopole solutions[44] had been constructed only in the "Prasad-Sommerfield limit,"[45] in which the Higgs fields which acquire vacuum expectation values and spontaneously break the gauge symmetry are exactly massless. In this limit, the Coulomb repulsion of the monopoles is exactly cancelled by the attractive force due to the exchange of the massless scalars, and the monopoles are noninteracting. It was expected that for any nonzero Higgs mass, monopoles would repel, and no stable bound states of monopoles would exist.

But it was recently discovered that stable multimonopole solutions do in fact exist.[46] The key observation is that monopoles with nonabelian long range magnetic fields can, in some cases, orient their magnetic charges in orthogonal directions in group space, and reduce their Coulomb repulsion to zero. In the Prasad-Sommerfield limit, the scalar-exchange force would also vanish for this relative charge orientation. But the scalar-exchange force is typically due to Higgs scalars in several different representations of the unbroken gauge group; when the charges are oriented so that there is no Coulomb repulsion, some of the Higgs scalars generate an attractive force and some a repulsive force. By choosing a Higgs scalar which generates an attractive force to be much lighter than the others, we can obtain an attractive interaction between the monopoles over a large range of separations. The result is a stable two-monopole bound state.

The above remarks apply to the SU(5) monopole if we recall that at separations much less than $M_{\overline{W}}^{-1}$, the monopole charges are free to choose orientations anywhere in SU(3) × SU(2) × U(1). Binding occurs if the SU(3) × SU(2) singlet member of the Higgs 24 multiplet is much lighter than the others. The existence of bound states of three, four, and six monopoles has also been demonstrated.[46]

Further work on the properties of these multimonopoles is desirable. In particular, their interactions with fermions should be investigated. Since some multimonopoles will not be spherically symmetric, and since W, Z fields are excited inside the multimonopoles, these interactions could have qualitatively new features.

Demise of Global Color

The dyonic charge excitations of the 't Hooft-Polyakov monopole arise when the global charge rotator degree of freedom of the monopole is quantized semiclassically. It was expected that semiclassical quantization of a monopole with a nonabelian "chromomagnetic" field would similarly give rise to a spectrum of "chromodyon" excitations in definite representations of the global color group. But early attempts to calculate the chromodyon spectrum of the SU(5) monopole uncovered some puzzles.[47]

Recently, it was discovered[48–49] that the only dyonic excitations of a chromomagnetic monopole which exist are those associated with the color rotations of the monopole which commute with its long range field; these "color hypercharge" excitations do not form complete color multiplets. Their failure to do so was clarified by the subsequent discovery[50] that a global color rotation of a monopole cannot even be implemented if the rotation does not commute with the monopole charge.

The surprising statement that global color rotations of a chromomagnetic monopole cannot be defined is better understood if we recall that global color is not a symmetry at all in the usual sense; physical states are gauge-invariant. Global color is important in monopole physics only as a means of keeping track of the collective coordinates of a monopole solution, for the purpose of semiclassical quantization.[51]

What has been found is that color rotations which act nontrivially on the long range chromomagnetic field are not acceptable as collective coordinates. Perhaps this is not really so surprising. The excitations associated with these rotations cannot be supported by the monopole core; they propagate to spatial infinity along the lines of magnetic force.

5. Monopoles and Fermions

The most spectacular recent developments in the theory of magnetic monopoles concern the interactions of monopoles and fermions. This subject has a complex history of which I will give a brief overview.

The recent developments have woven together several independent lines of inquiry into the properties of monopoles. First, an analysis[52] of the quantum mechanics of a charged spin-$\frac{1}{2}$ particle in the field of a point monopole led Goldhaber[53] to observe that this problem is inherently ambiguous; the behavior of an electron scattering off a point monopole cannot be uniquely determined until a boundary condition is chosen for the electron wave function at the location of the magnetic pole.

Later, Dokos and Tomaras[54] pointed out that a magnetic monopole in a grand unified theory can catalyze processes which change baryon number. They noted that the dyonic excitations of the SU(5) monopole have baryon-number-violating couplings, and that a collision which excites the dyon degree of freedom need not conserve baryon number. But they believed that the lightest dyon was split from the monopole ground state by an amount of order $\alpha M_X \gtrsim 10^{13}$ GeV, so that the catalysis cross section, suppressed by a huge energy denominator, was extremely small.

Meanwhile, Witten[39] discovered that the monopole ground state carries the anomalous electric charge $Q = e\vartheta/2\pi$, where ϑ, the "vacuum angle,"[55] is an arbitrary parameter defined modulo 2π. Witten's discovery, like Goldhaber's, suggested that the dynamics of a fermion coupled to a monopole is rather subtle, for it is known that, because of the axial anomaly,[56] the angle ϑ becomes unobservable if there are massless charged fermions.[56] The question arose, what happens to the anomalous charge of the monopole as m_e, the electron mass, approaches zero?

This issue was clarified by Blaer et al.[6], Wilczek[7], Rubakov[8], ánd Callan[9], who concluded that the electric charge of the monopole is smeared out over a region with radius of order m_e^{-1}; thus, the anomalous charge disappears in the limit $m_e = 0$. It follows that the dyon is actually split from the monopole ground state by a tiny amount of order αm_e, and is easily excited. Moreover, Wilczek[7] and Rubakov[8] emphasized that, because of the axial anomaly[56], the monopole is not an eigenstate of chirality or baryon number. This observation opened the possibility of a large chirality-violating, and, perhaps, baryon-number-violating cross section in monopole-fermion scattering.

Callan[9] had meanwhile developed a different perspective on the catalysis process. Besson[57] and he recognized that the boundary condition needed to completely specify the physics of a fermion scattering off a point monopole could be obtained by considering the case of a nonsingular monopole, and then taking the limit of vanishing monopole core size. This procedure led Callan to the remarkable conclusion that the physics of the monopole core need not "decouple" from low energy fermion-monopole scattering. In particular, baryon-number-violating interactions inside the core can induce baryon-number-changing scattering processes with a cross section unsuppressed by the exceedingly small core size.

Thus, two different explanations of the catalysis phenomenon emerged. One is that the catalysis of baryon-number-changing reactions by monopoles is a consequence of an anomaly afflicting the baryon number current. The other is that catalysis is a consequence of a baryon-number-violating boundary condition reflecting the physics of the monopole core. The two explanations are sometimes confused, but cannot, in general, be regarded as complementary descriptions of the same phenomenon.[58–60] By the "Callan-Rubakov effect," one usually means symmetry violation arising in the form of a boundary condition at the monopole core.

To begin to understand the Callan-Rubakov phenomenon, recall that if a particle of electric charge e moves in the field of a point monopole with magnetic charge g, the electromagnetic field carries angular momentum

$$\vec{J}_{em} = e g \hat{r} , \tag{5.1}$$

where $\hat{r}$ is the unit vector pointing toward the charged particle from the monopole.[11] If the charged particle were to pass through the monopole, this contribution to the angular momentum would change discontinuously. Therefore, conservation of angular moment forbids the particle to pass through the pole, unless its charge or intrinsic spin can change instantaneously as it does so.

The above remark has a quantum mechanical counterpart, as we see[52-53] by solving the Dirac equation for a massless electron in the field of a point monopole with $eg = \frac{1}{2}$. The $J = 0$ solutions to the Dirac equation are peculiar; the positive helicity solution is purely an outgoing wave, and the negative helicity solution is purely an incoming wave. (For a positron, the helicities of the solutions are reversed.) Both solutions are singular at the origin, the location of the pole, and the Dirac equation itself provides no criterion for matching up the incoming and outgoing solutions. The Hamiltonian defined by the Dirac equation is therefore not self-adjoint; probability is not conserved unless the Hamiltonian is supplemented by a boundary condition which relates the incoming and outgoing waves.

This peculiar behavior is not too hard to understand. An electron in the monopole field has $\vec{J}_{em} = -\frac{1}{2}\hat{r}$. Hence, an incoming (outgoing) electron must have negative (positive) helicity to be in a state with $\vec{J} = \vec{J}_{em} + \vec{\sigma} = 0$. For a positron, $\vec{J}_{em}$ has the opposite sign, and the helicities are reversed.

The boundary condition at the origin determines the fate of a left-handed electron which scatters off a monopole in the $J = 0$ partial wave. But there are only two options; it becomes either a right-handed electron or a left-handed positron, because those are the only available outgoing modes with $J = 0$. The boundary condition must therefore either violate chirality (which is otherwise a good symmetry of the Hamiltonian) or require the monopole to absorb electric charge. If the charge-conserving boundary condition is chosen, then the chirality-changing $J = 0$ cross section will saturate the unitarity limit.

That a boundary condition must be specified to determine the final state of an electron scattering off a point monopole is the key to the Callan-Rubakov effect. What makes the effect profoundly disturbing is that it seems to violate a fundamental principle of quantum field theory, the decoupling principle,[61] which asserts that the effects of very short distance physics must be power suppressed at low energy. The decoupling principle leads one to expect that the amplitude for monopole-fermion scattering at energies much less than the inverse size of the monopole core will not depend on the structure of the core, except for power corrections which vanish as the size of the core goes to zero. This expectation fails because of the ambiguity in monopole-fermion scattering when the monopole is pointlike; information about the structure of the core survives at low energy in the form of a boundary condition which exerts a strong influence on low-energy physics. In particular, the boundary conditions may violate a symmetry (like baryon number) which would otherwise be a good symmetry of the effective theory describing low-energy physics.

The analysis of the scattering of a low-energy fermion off a nonsingular monopole can be divided into two parts. First, we decide what boundary conditions to impose as the limit of vanishing core size is taken. Then the interaction of a point monopole with fermions satisfying the appropriate boundary conditions is studied. The second step is highly nontrivial. Fermion pair creation effects, which are responsible for smearing out the dyon charge over a region of order the fermion Compton wavelength, must be taken into account as fully as possible. Both Rubakov[8,62] and Callan[9,63] suggested that, since only $J = 0$ fermions can penetrate to the core of the monopole, the problem can be well-approximated by an effective $(1 + 1)$-dimensional quantum field theory describing the $J = 0$ partial wave. The qualitative feature of this $(1 + 1)$-dimensional theory are most easily glimpsed if it is converted into an equivalent "bosonized" theory[64] in which the fermions are represented by solitons. The soliton picture of monopole-fermion scattering is especially convenient when we try to understand the effects of fermion masses.

Returning to the problem of finding the appropriate boundary conditions satisfied by the fermions, let us consider, for concreteness, the case of the SU(5) model with a single generation of fermions[65]. The magnetic charge of the SU(5) monopole is actually a linear combination of ordinary magnetic charge and color magnetic charge. At distances from the monopole center much less than $10^{-13} cm$ and much greater than the radius of the core, the only fermions which interact with the monopole are those which carry Q, the corresponding combination of electric charge and color electric charge. The right-handed quarks and leptons with nonzero Q are, in an appropriate gauge,

$$\begin{array}{ll} Q = 1: & e_R^+ \; \bar{d}_{3R} \; u_{1R} \; u_{2R} \text{ (incoming)} \\ Q = -1: & d_{3R} \; e_R^- \; \bar{u}_{2R} \; \bar{u}_{1R} \text{ (outgoing)}, \end{array} \tag{5.2}$$

where 1, 2, 3 are color indices. The behavior of these fermions in the field of the SU(5) monopole is identical to the behavior of an electron or positron in the field of an ordinary Dirac monopole. The only new feature is that there are four

(Dirac) fermions interacting with the monopole, and the boundary condition at the origin causes these fermions to mix in a manner determined by the structure of the core of the monopole.

One can attempt to determine the boundary condition by solving the Dirac equation in the field of the nonsingular SU(5) monopole with finite core radius.[66,57,9] The result is that the helicity of the incoming fermion is preserved; incoming and outgoing states are matched up as in (5.2). We see that two units of Q are transferred to the monopole, exciting its dyon degree of freedom.

But if we now investigate the consequences of this boundary condition, taking proper account of pair creation effects, we realize that the picture in which the incoming fermion falls to the core and deposits charge there, suggested by the solution to the Dirac equation, is not very accurate. An enormous Coulomb barrier prevents charge from being deposited on the core. It is energetically favored for the charge to be spread out over a region with a radius of order a fermion Compton wavelength. As a result, our original procedure for finding the correct boundary condition is called into question. It seems that a more appropriate boundary condition is one that forbids charge to accumulate at the origin.[67]

Fortunately and remarkably, in the case of the SU(5) monopole we can obtain quite nontrivial information about the scattering process by merely demanding that none of the charges coupling to massless gauge bosons accumulate on the core.[68] This constraint is especially interesting because the W and Z bosons must be regarded as effectively massless at distances from the center of the monopole much less than M_W^{-1}. Since left-handed and right-handed fermions with the same electric charge have different values of $(T_3)_{\mathrm{weak}}$, simple chirality violating processes such as

$$e_L^- + M \to e_R^- + M \tag{5.3}$$

are forbidden for massless fermions! If, for example, two u-quarks scatter off the monopole, there is only one possible two-fermion final state; the allowed process is

$$u_{1R}\, u_{2R} + M \rightarrow \bar{d}_{3L}\, e^+_L + M \ . \tag{5.4}$$

Baryon number violation is forced on us, if we ignore the masses of the fermions. All we need to know about the SU(5) model is that it contains a monopole which couples to the charge Q given by (5.2), in a phase with unbroken $SU(3)_c \times [SU(2) \times U(1)]_{ew}$.

The process (5.4), with two fermions in the initial state, must occur in two steps. It is natural to inquire about the intermediate state produced when u_{1R} scatters off the monopole. What one finds[58-59,69] is rather subtle and mysterious; the intermediate state consists of four "semitons," each with fermion number 1/2. The reaction

$$u_{1R} + M \rightarrow \tfrac{1}{2}(u_{1L}\, \bar{u}_{2R}\, \bar{d}_{3L}\, e^+_L) + M \tag{5.5}$$

changes baryon number by $-\frac{1}{2}$ unit.

The semitons are destabilized by fermion mass terms or the effects of the strong QCD interaction. At a distance from the monopole center where these effects become important, the intermediate state in (5.5) evolves into a final state with baryon and lepton number differing by an integer from that of the initial state. One possibility is that the semitons in (5.5) evolve into u_{1L}; chirality violating processes like (5.3) are allowed if the fermions have masses.

The evolution of semitons into final state quarks and leptons can be studied numerically by integrating the classical equations of motion of the bosonized version of the effective (1 + 1)-dimensional field theory, in which the fermions are represented by solitons. The classical approximation cannot be justified in detail, but is expected to give qualitatively correct results. Unsurprisingly, the semiton intermediate state is found[59] to evolve into a final state with a different baryon number than the initial state with a probability of order one. It is also found that adding more generations of fermions has no qualitative effect on the baryon-number-changing processes.

The above considerations strongly suggest that the baryon-number-changing cross section for a quark of energy E scattering off a monopole is of order E^{-2}, if E^{-1} is much greater than the radius of the monopole core, and much less than both the Compton wavelength of the quark and the size of a hadron. But we are really interested in the cross section of *nucleon* decay. There are actually two questions of experimental interest. One is, what is the cross section for capture of nuclei by the monopole? It is presumably large[70], and the capture rate might conceivably determine the catalysis rate in terrestrial experiments. The other is, what is the cross section for the catalysis process itself? In particular, is it much smaller the the naive guess $\sigma\beta \sim 10^{-27} cm^2$?

A first step toward answering the latter question can be taken within the context of an approximation in which the nucleon is regarded as a soliton in a nonlinear effective chiral field theory.[71] This approximation should be reasonable at distances large compared to the confinement scale, and so is in a sense complementary to the quark model description of the nucleon. It is found[72] that a chiral soliton which encounters a point monopole can "unwind" and decay, if an appropriate boundary condition is satisfied at the pole. Preliminary calculations thus suggest that the catalysis of nucleon decay by monopoles is not significantly suppressed by nonperturbative strong interaction effects.

Our understanding of the catalysis of nucleon decay by monopoles is still far from complete. For one thing, the production and evolution of semitons could probably be elucidated by further work. Also, the effects of the nondiagonal color and weak charges on the scattering process have not been analyzed.[73]

It might be possible to use the chiral soliton approach to obtain some qualitative information about the velocity dependence and multiplicities in catalyzed nucleon decay, although the corrections to this approximation are expected to be of order one. Any other new ideas about how cross sections might be calculated would be welcome.

6. Conclusions.

I have attempted to review only a few of the recent developments in theoretical physics which concern magnetic monopoles. Various matters deserving further investigation have been noted, such as monopole production in the inflationary cosmology, the properties of Kaluza-Klein monopoles and multimonopole bound states, and the rates of processes catalyzed by monopoles.

The magnetic monopole has proved to be a rich source of theoretical speculation and discovery. On the basis of an eyeball extrapolation of Fig. 1a, it seems reasonable to conjecture that the monopole will continue to command our attention for some time to come.

References

1. SPIRES-3, SLAC library data base (1983).

2. P. B. Price et al., Phys. Rev. Lett. 35, 487 (1975).

3. B. Cabrera, Phys. Rev. Lett. 48, 1378 (1982).

4. G. 't Hooft, Nucl. Phys. B79, 276 (1974).

5. A. M. Polyakov, JETP Lett. 20, 194 (1974).

6. A. Blaer, N. Christ, and J. Tang, Phys. Rev. Lett. 47, 1364 (1981).

7. F. Wilczek, Phys. Rev. Lett. 48, 1146 (1982).

8. V. Rubakov, JETP Lett. 33, 644 (1981); Nucl. Phys. B203, 311 (1982).

9. C. G. Callan, Phys. Rev. D25, 2141 (1982); Phys. Rev. D26, 2058 (1982).

10. E. Weinberg, these proceedings.

11. S. Coleman, in *The Unity of the Fundamental Interactions*, A Zichichi, New York, Plenum (1983).

12. M. Turner, these proceedings.

13. J. Preskill, in *The Very Early Universe*, ed. G. W. Gibbons, S. W. Hawking, and S. T. C. Siklos, New York, Cambridge (1983).

14. J. Preskill, Phys. Rev. Lett. 43, 1365 (1979).

15. M. S. Turner, Phys. Lett. 115B, 95 (1982).

16. G. Lazarides, Q. Shafi, and W. P. Trower, Phys. Rev. Lett. 49, 1756 (1982).

17. P. Langacker and S. -Y. Pi, Phys. Rev. Lett. 45, 1 (1980).

18. F. A. Bais and P. Langacker, Nucl. Phys. B197, 520 (1982);

A. Vilenkin, Nucl. Phys. B196, 240 (1982).

19. E. Weinberg, Phys. Lett. 126B, 441 (1983).

20. M. S. Turner, these proceedings;

W. Collins and M. S. Turner, Fermi Inst. Preprint 83-41 (1983).

21. M. S. Turner, unpublished;

A. S. Goldhaber, A. H. Guth, and S. -Y. Pi, unpublished.

22. M. S. Turner, in *Magnetic Monopoles*, ed. R. A. Carrigan and W. D. Trower, New York, Plenum (1983);

these proceedings.

23. E. N. Parker, Ap. J. 160, 383 (1970).

24. M. S. Turner, E. N. Parker, and T. J. Bogdan, Phys. Rev. D26, 1296 (1982).

25. R. A. Carrigan and W. D. Trower, *Magnetic Monopoles*, New York, Plenum (1983).

26. S. Dimopoulos, S. L. Glashow, E. M. Purcell, and F. Wilczek, Nature 298, 824 (1982).

27. K. Freese and M. S. Turner, Phys. Lett. 123B, 293 (1983).

28. E. E. Salpeter, S. L. Shapiro, and I. Wasserman, Phys. Rev. Lett. 49, 1114 (1982).

29. E. W. Kolb, S. A. Colgate and J. Harvey, Phys. Rev. Lett. 49, 1373 (1982);

S. Dimopoulos, J. Preskill, and F. Wilczek, Phys. Lett. 119B, 320 (1982);

E. W. Kolb, these proceedings.

30. J. Harvey, these proceedings.

31. S. Dawson and A. N. Schellekens, Phys. Rev. D27, 2119 (1983).

32. J. Harvey, Princeton Preprint Print-82-0868 (1982).

33. K. Freese, these proceedings.

34. B. Julia and A. Zee, Phys. Rev. D11, 2227 (1975);
E. Tomboulis and G. Woo, Nucl. Phys. B107, 221 (1976).

35. R. Jackiw and C. Rebbi, Phys. Rev. Lett. 36, 1116 (1976);
P. Hassenfratz and G. 't Hooft, Phys. Rev. Lett. 36, 1119 (1976);
A. Goldhaber, Phys. Rev. Lett. 36, 1122 (1976).

36. G. 't Hooft, Nucl. Phys. B105, 538 (1976),
E. Corrigan, D. Olive, D. Fairlie and J. Nuyts, Nucl. Phys. B106, 475 (1976).

37. G. 't Hooft, Nucl. Phys. B153, 141 (1979).

38. S. Mandelstam, Phys. Rep. 23C, 235 (1976);
G. 't Hooft, Nucl. Phys. B190, 455 (1981).

39. E. Witten, Phys. Lett. 86B, 283 (1979).

40. R. Jackiw and C. Rebbi, Phys. Rev. D13, 3398 (1976);
J. Goldstone and F. Wilczek, Phys. Rev. Lett. 47, 986 (1981).

41. Th. Kaluza, Sitzungsber. Preuss. Akad. Wiss. Math. Phys. 1921, 966 (1921);
O. Klein, Z. Phys. 37, 895 (1926).

42. D. Gross and M. Perry, Nucl. Phys. B226, 29 (1983);
R. Sorkin, Phys. Rev. Lett. 51, 87 (1983).

43. P. Nelson, Harvard Preprint HUTP-83/A074 (1983).

44. E. Weinberg, Phys. Rev. D20, 936 (1979);
Nucl. Phys. B167, 500 (1980).

45. M. Prasad and C. Sommerfield, Phys. Rev. Lett. 35, 760 (1975);
E. Bogomol'nyi, Sov. J. Nucl. Phys. 24, 449 (1976);
S. Coleman, S. Parke, A. Neveu, and C. Sommerfield, Phys. Rev. D15, 544 (1977).

46. C. L. Gardner, J. A. Harvey, Princeton Preprint Print-83-1062 (1983).

47. C. Dokos and T. Tomaras, Phys. Rev. D21, 2940 (1980).

48. A. Abouelsaood, Nucl. Phys. B226, 309 (1984);
Phys. Lett. 125B, 467 (1983).

49. P. Nelson, Phys. Rev. Lett. 50, 939 (1983).

50. P. Nelson and A. Manohar, Phys. Rev. Lett. 50, 943 (1983);
A. Balachandran et al. Phys. Rev. Lett. 50, 1553 (1983).

51. P. Nelson and S. Coleman, Harvard Preprint HUTP-83/A067 (1983).

52. Y. Kazama, C. N. Yang, and A. S. Goldhaber, Phys. Rev. D15, 2287 (1977).

53. A. S. Goldhaber, Phys. Rev. D16, 1815 (1977).

54. C. Dokos and T. Tomaras, Phys. Rev. D21, 2940 (1980).

55. G. 't Hooft, Phys. Rev. Lett. 37, 8 (1976);
C. G. Callan, R. F. Dashen and D. J. Gross, Phys. Lett. 63B, 334 (1976);
R. Jackiw and C. Rebbi, Phys. Rev. Lett. 37, 177 (1976).

56. J. S. Bell and R. Jackiw, Nuovo Cim 51, 47 (1969);
S. L. Adler, Phys. Rev. 177, 2426 (1969).

57. C. Besson, Ph.D. Thesis, Princeton University (1982).

58. A. Sen, Fermilab-Pub-83/28 (1983);
these proceedings.

59. S. Dawson and A. N. Schellekens, Fermilab-Pub-83/43-THY (1983).

60. Z. F. Ezawa and A. Iwazaki, preprint (1983).

61. T. Appelquist and J. Carazzone, Phys. Rev. D11, 2856 (1975).

62. V. A. Rubakov and M. S. Serbryakov, Nucl. Phys. B218, 240 (1983).

63. C. G. Callan, Nucl. Phys. B212, 391 (1983).

64. S. Coleman, Phys. Rev. D11, 2088 (1975);
S. Mandelstam, Phys. Rev. D11, 3026 (1974).

65. H. Georgi and S. L. Glashow, Phys. Rev. Lett. 32, 438 (1974).

66. R. Jackiw and C. Rebbi, Phys. Rev. D13, 3398 (1976).

67. Y. Kazama and A. Sen, Fermilab-Pub-83/58 (1983);
T. -M. Yan, Cornell Preprint CLNS-83/563 (1983);
A. P. Balachandran and J. Schechter, Syracuse Preprint (1983);
Z. F. Ezawa and A. Iwazaki, Preprint (1983);
H. Yamagishi, Princeton Preprint (1983).

68. B. Grossman, G. Lazarides, and A. I. Sanda, Phys. Rev. D28, 2109 (1983).

69. C. G. Callan, Princeton Preprint 83-0308 (1983).

70. G. Fiorentini, these proceedings.

71. T. Skyrme, Proc. Roy. Soc. A260, 127 (1961);
Nucl. Phys. 31, 556 (1962);
E. Witten, Nucl. Phys. B223 433 (1983).

72. C. G. Callan and E. Witten, Princeton Preprint Print-84-0054 (1983);
C. G. Callan, these proceedings.

73. N. S. Craigie, W. Nahm, and V. A. Rubakov, Preprint (1983).

Figure Captions

1. Interest in magnetic monopoles as a function of time. (a) overall. (b) experimental. (c) field theory. (d) cosmology and astrophysics. (e) interactions with fermions.

2. Astrophysical limits on the monopole flux as a function of monopole mass. "Galactic B field" labels the limit based on the energetics of the galactic magnetic field. "Galactic mass" labels the limit based on the total mass of the galaxy.

3. Magnetic monopole in a flux tube near the boundary between a proton pair condensate and a charged pion condensate. (a) In the proton pair condensate, the magnetic flux in the tube reverses direction at the monopole, and there is no net magnetic force on the monopole. (b) In the pion condensate, the flux tube terminates on the monopole, and the monopole accelerates rapidly.

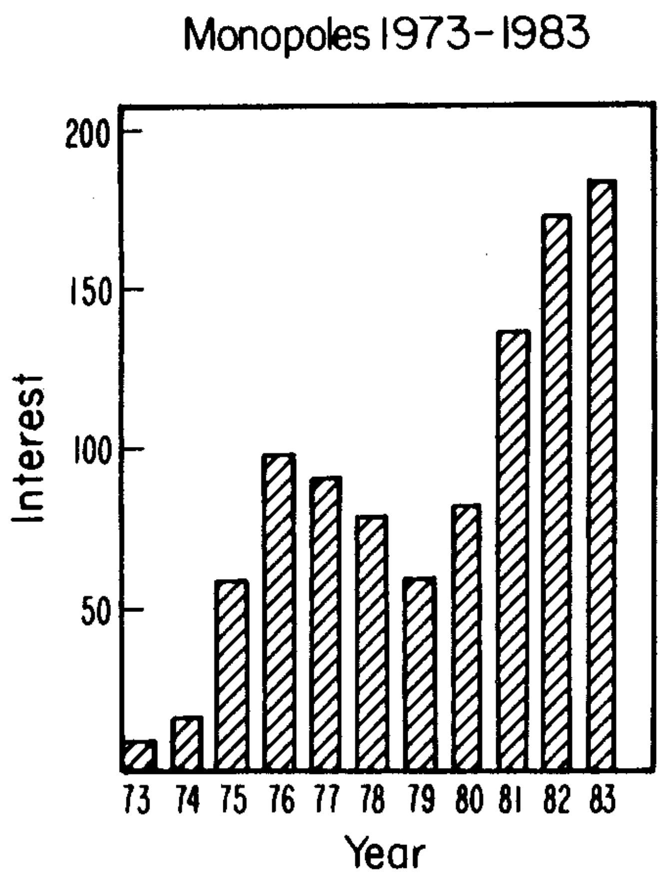

Monopoles 1973-1983
Interest
200
150
100
50
73 74 75 76 77 78 79 80 81 82 83
Year

Fig. 1a

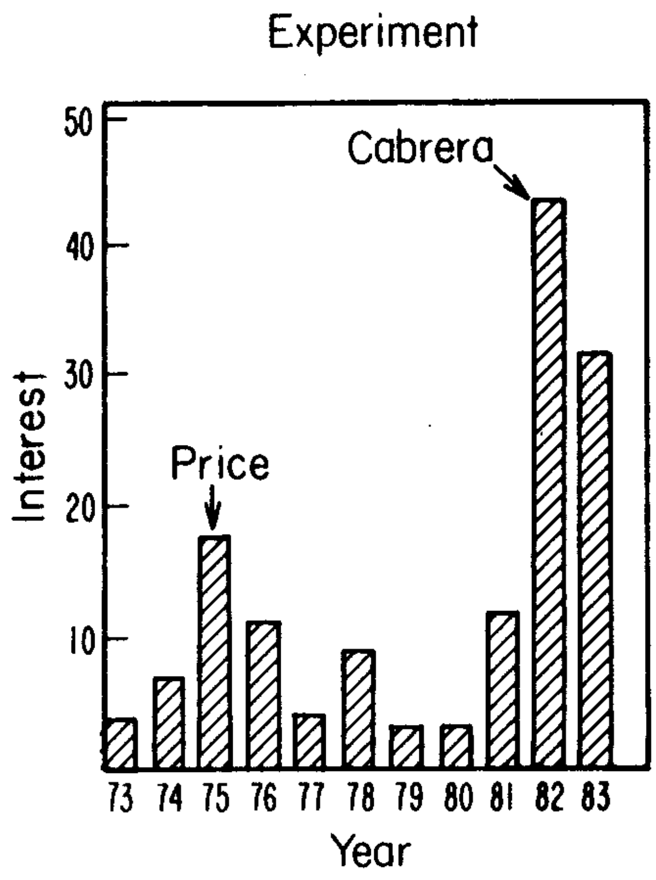
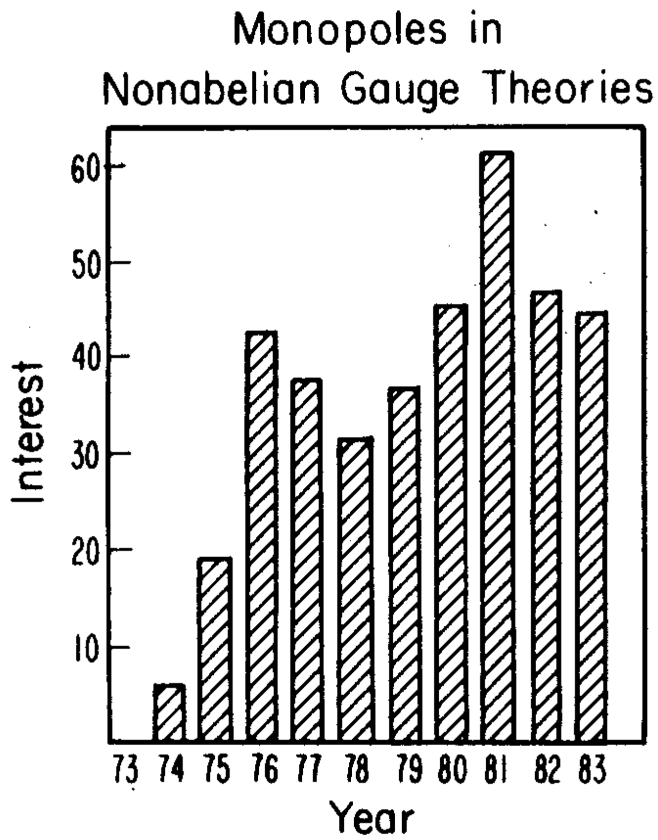

Experiment
Interest
50
40
30
20
10
Price
Cabrera
73 74 75 76 77 78 79 80 81 82 83
Year

Fig. 1b


Monopoles in
Nonabelian Gauge Theories
Interest
60
50
40
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20
10
73 74 75 76 77 78 79 80 81 82 83
Year

Fig. 1c

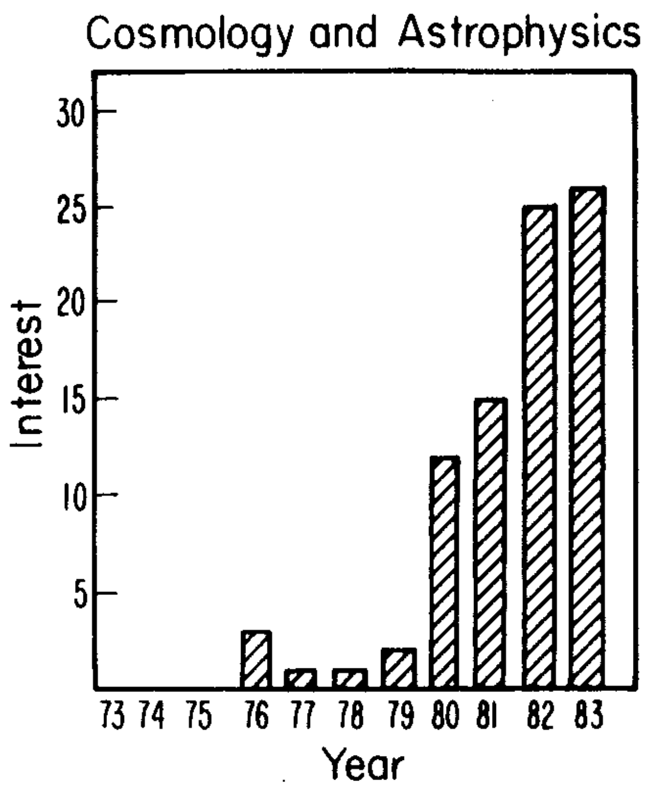

Cosmology and Astrophysics
Interest
30
25
20
15
10
5
73 74 75 76 77 78 79 80 81 82 83
Year

Fig. 1d

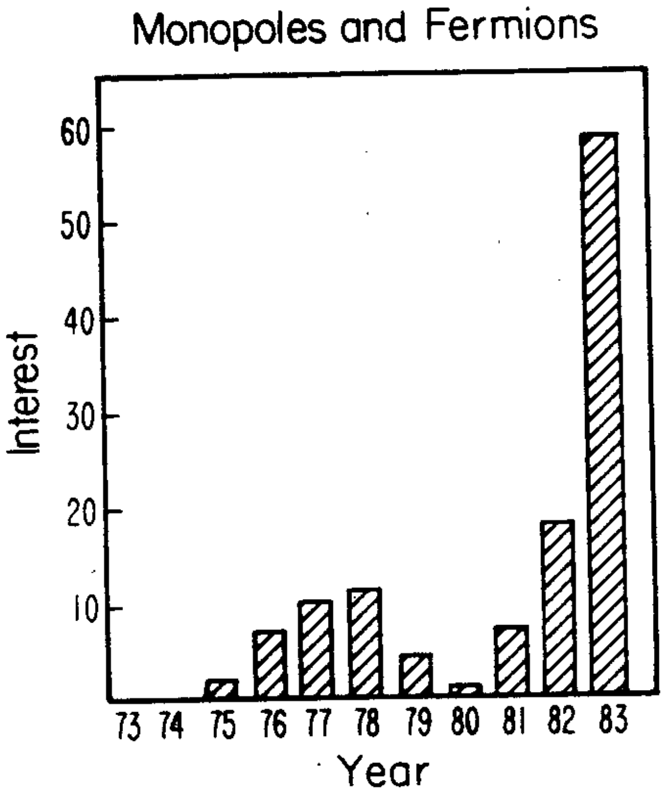
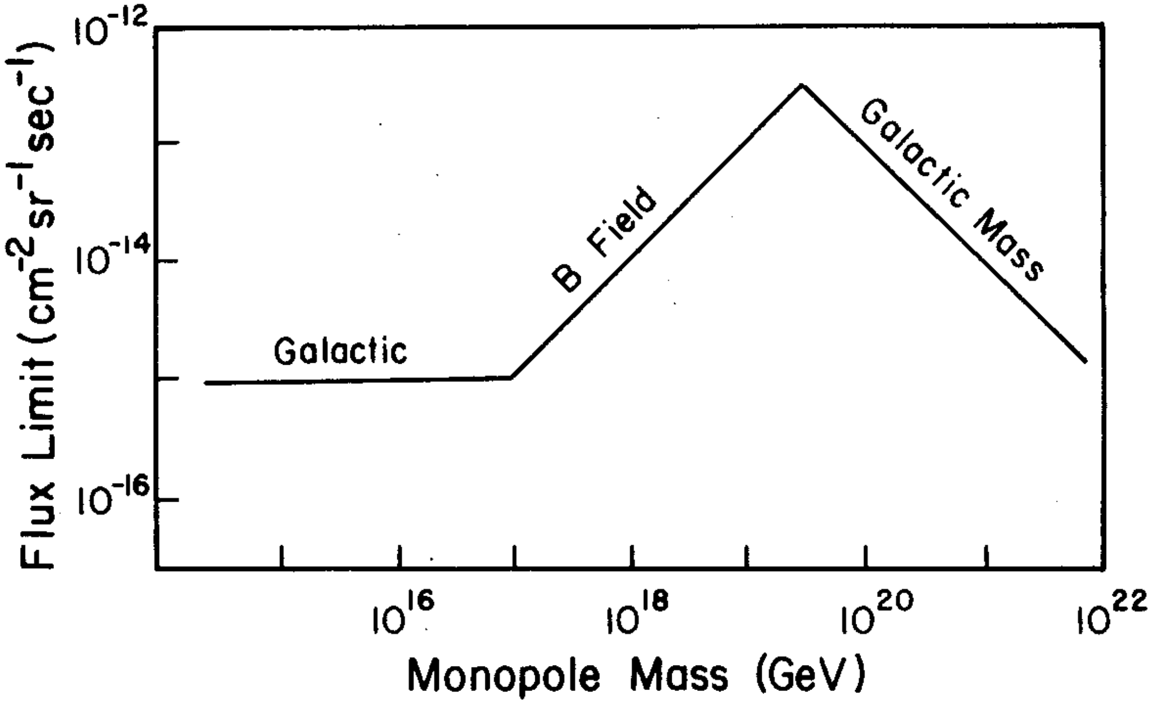

Monopoles and Fermions
60
50
40
30
20
10
Interest
73 74 75 76 77 78 79 80 81 82 83
Year

Fig. 1e



Flux Limit (cm^-2 sr^-1 sec^-1)
10^-12
10^-14
10^-16
Galactic
B Field
Galactic Mass
10^16
10^18
10^20
10^22
Monopole Mass (GeV)

Fig. 2

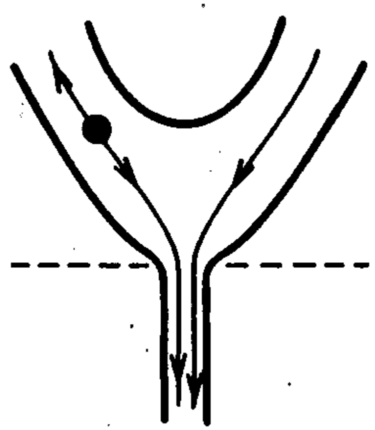


Fig. 3a

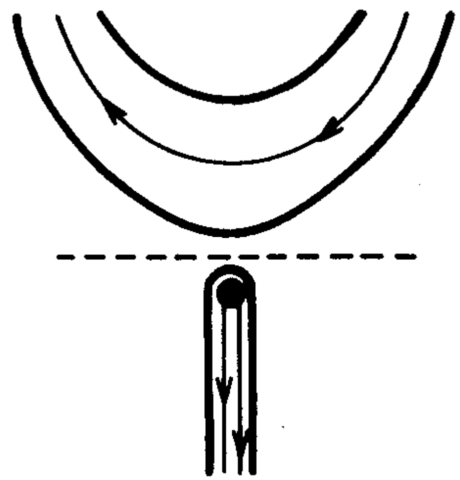


Fig. 3b