COMPOSITE FERMIONS WITHOUT U(1)'s

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We discuss a class of models that may produce massless composite fermions which carry nonabelian unbroken chiral symmetries, but no chiral U(1) symmetries. We analyze these models both in the confining picture and in a variant of the most-attractive-channel picture.

In a recent paper [1], one of us suggested that the composite Higgs mechanism [2] could be combined with models in which composite fermions are kept light by unbroken global chiral symmetries [3] to produce realistic models of the strong and electroweak interactions of quarks and leptons. A particularly interesting class of models was discussed in which the global symmetries that require the masslessness of the Higgs doublet and the quarks and leptons are broken only by weak gauge couplings (like SU(2) × U(1)). It was shown that in all such models, while the W and Z and Higgs boson masses are of order gv (where g is a typical electroweak gauge coupling and v is the SU(2) × U(1) breaking VEV of the Higgs doublet), the quark and lepton masses are at most of order g^2v. Furthermore, a mechanism which leads to a hierarchy of quark and lepton masses (related to powers of gauge couplings) was exhibited. In ref. [1], the composite models were studied using the techniques of effective lagrangians [4] and chiral perturbative theory [5], which depend only on the symmetry structure. In particular, it was not necessary to discuss the structure of the theory above the confinement scale. However, there are interesting questions which cannot be addressed in this language. The most important is the question of whether models with the properties discussed in ref. [1] exist at all. But in addition, once such models have been explicitly constructed above the confinement scale, we can begin to discuss such amusing issues as grand unification.

On the other hand, it is not straightforward to build such models because the dynamics of strongly interacting gauge theories with chiral fermions is not well understood. One necessary condition for the existence of such theories is well known: the light fermions which survive below the confinement scale must satisfy 't Hooft's anomaly condition [3] and reproduce the anomalies of the unbroken subgroup of the original theory. However, it is also well known that this condition is not sufficient. In addition to the anomaly condition, we would like to have a plausible dynamical picture which suggests that chiral symmetries actually remain unbroken and that massless fermions are actually produced.

There are several well-known classes of models which seem very likely to produce chiral fermions.

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Probably the most convincing are models in which the strong gauge group is SU(N) and the left-handed (LH) fermions transform as a two-index antisymmetric tensor ($N(N-1)/2$) and $(N-4)\bar{N}$'s [6,7]. In a most-attractive-channel (MAC) picture, the gauge group breaks down to SU(4) and the global SU($N-4$) × U(1) symmetry of the original theory combines with a subgroup of the SU(N) to leave a global SU($N-4$) × U(1) unbroken. Under this symmetry there are fermions (just pieces of the original fermion representation) that are gauge SU(4) singlets and transform under the global SU($N-4$) as a two-index symmetric tensor. These fermions must remain massless because of the unbroken chiral symmetry. The same conclusions can be reached in the "complementary" confining picture [7], in which the SU(N) gauge symmetry is assumed to remain unbroken and the chiral fermions are interpreted as composite states. Indeed, it is, in part, the congruence of these two different pictures that gives us confidence in our analysis of these models.

It is easy to extend such models such that the same analysis suggests that composite Goldstone bosons as well as chiral fermions are formed. Simply add MN's and $M\bar{N}$'s. Now the global symmetry is SU($N+M-4$) × SU(M) × U(1) × U(1) and the MAC picture suggests that it breaks down to SU($N-4$) × SU(M) × U(1) × U(1), producing the same chiral fermions and Goldstone bosons in the manifold SU($N+M-4$)/SU($N-4$) [‡1]. Unfortunately, such models are not suitable for our purposes because the chiral fermions carry an unbroken chiral U(1) symmetry which is not carried by the composite Higgs. Thus Yukawa couplings of the composite Higgs to fermions cannot be produced, even in the presence of weak gauge couplings because weak gauge boson exchange does not break the chiral U(1) symmetry.

In this note, we suggest and discuss a very different class of models that, we believe, is equally likely to lead to chiral, composite, fermions and in which the chiral fermions are protected by unbroken non-abelian chiral symmetries, but not by any U(1)'s. In a separate note, we will show that Yukawa couplings between the composite Higgs and the composite fermions may actually be induced in models of this kind [9].

We begin by constructing a class of models in which there are chiral fermions but no composite Higgs. In order to avoid U(1)'s which act on the chiral fermions, we will avoid U(1)'s entirely by starting with a semisimple strong interaction in which there are as many simple factors as there are types of fermion representations. The simplest interesting example is the following: The gauge group is SU(L) × SU(M) × SU(N). The LH fermions are

$$\begin{aligned} &\mathrm{L}(1, M, \bar{N})\text{'s} \quad (\text{type L}), \\ &\mathrm{M}(\bar{L}, 1, N)\text{'s} \quad (\text{type M}), \\ &\mathrm{N}(L, \bar{M}, 1)\text{'s} \quad (\text{type N}). \end{aligned} \tag{1}$$

Evidently, there is an su(L) × su(M) × su(N) global symmetry [‡2] because of the identical fermion representations, but all of the possible global U(1) symmetries are anomalous with respect to at least one of the strong groups. Note, however, that the model has no gauge anomalies and that, for suitable choices of the integers L, M, and N, all the gauge couplings are asymptotically free [‡3]. We further assume that they all get strong at about the same scale, the confinement scale.

If we assume that the su(L) × su(M) × su(N) global symmetries are unbroken by the confinement mechanism, there is an extremely simple way of saturating the anomaly conditions. We can assume that the massless LH composite fermions transform under su(L) × SU(M) × SU(N) as an (L, M, N). Furthermore, this is just what is suggested by a very simple confinement picture in which the composite fermions are gauge singlet bound states of three elementary fermions, one each of type L, type M, and type N.

The models in (1) are interesting because they look like chiral fermion models only when two or more of the simple gauge subgroups get strong at once. If only one of the gauge subgroups gets strong, the model resembles QCD with some of the global chiral symmetries weakly gauged and the resulting anomalies canceled by spectator fermions. Thus, even though we are really interested in the situation in which all the gauge subgroups get strong at once, it may be possible

‡1 Such models have also been discussed by Bars and Yankielowicz [8], but they assume that all the global chiral symmetries remain unbroken.

‡2 Here and below, we use lower case letters (i.e. su(M)) to refer to the global symmetry groups.

‡3 The condition is: $2LMN < 11\,[\min(L, M, N)]^2$.

to get some insight into the dynamics by treating one of the gauge subgroups, say SU(L), as a QCD-like confining interaction, and the other two as "weak" gauge interactions which break the global chiral symmetries.

Only the type M and N fermions in (1) transform nontrivially under SU(L), so the global chiral symmetries of the SU(L) theory in the absence of SU(M) and SU(N) interactions would be su(MN) × su(MN) × u(1), spontaneously broken down to su(MN) × u(1). In the presence of the chiral symmetry breaking produced by the SU(M) × SU(N) gauge interactions, there is a nontrivial vacuum alignment question: how do the two gauge groups line up with one another and with the two relevant global symmetry groups su(N) and su(M)?

Call the type M fermions

$$\psi_{\mathrm{M}}^{aj} \quad \text{for } a = 1, ..., M, j = 1, ..., N\,, \tag{2}$$

suppressing the gauged SU(L) indices. Call the type N fermions

$$\psi_{ja}^{\mathrm{N}} \quad \text{for } j = 1, ..., N, a = 1, ..., M\,. \tag{3}$$

The condensate which describes the spontaneous breaking of the chiral symmetry can then be represented by a unitary $MN \times MN$ matrix

$$\Sigma^{aj}{}_{kb} \propto \langle \psi_{\mathrm{M}}^{aj} \psi_{kb}^{\mathrm{N}} \rangle\,. \tag{4}$$

On this condensate, the generators of the various gauge and global symmetries act as follows:

$$\text{global su}(M) \to (T_\alpha^{\mathrm{M}})^a_c \Sigma^{cj}{}_{kb}\,,$$

$$\text{global su}(N) \to \Sigma^{aj}{}_{lb} (T_\mu^{\mathrm{N}*})^l_k\,,$$

$$\text{gauge SU}(N) \to (T_\mu^{\mathrm{N}})^j_l \Sigma^{al}{}_{kb}\,,$$

$$\text{gauge SU}(M) \to -\Sigma^{aj}{}_{kc} (T_\alpha^{\mathrm{M}})^c_b\,, \tag{5}$$

where

$$T_\alpha^{\mathrm{M}} \quad \text{for } \alpha = 1, ..., M^2 - 1\,,$$

$$T_\mu^{\mathrm{N}} \quad \text{for } \mu = 1, ..., N^2 - 1\,, \tag{6}$$

are $M \times M$ and $N \times N$ generalizations of the Gell-Mann matrices.

We can now discuss the vacuum alignment question by constructing an effective lagrangian which depends on the Σ field. In this case, however, the order g^2 effective potential does not determine the vacuum alignment. Because the M type and N type fermions do not share any gauge symmetry, the order g^2 potential is independent of Σ. But the physics does depend on Σ through the Σ dependence of the M and N gauge boson masses. This is the situation in which gauge boson loops are important in determining the vacuum alignment through the Coleman–Weinberg mechanism [10].

The gauge boson masses in order g^2 come from the Goldstone boson kinetic energy term

$$\tfrac{1}{4} f_L^2 \, \mathrm{tr}\, \mathrm{D}^\mu \Sigma \mathrm{D}_\mu \Sigma^\dagger\,, \tag{7}$$

where f_L is the analog of f_π for the SU(L) strong interactions. In the ($M^2 + N^2 - 2$)-dimensional space of the SU(M) and SU(N) gauge bosons, the gauge boson mass-squared matrix is

$$M_{\alpha\beta}^2 = \tfrac{1}{2} g_M^2 f_L^2 \, \mathrm{tr}\, T_\alpha^{\mathrm{M}} T_\beta^{\mathrm{M}} = \tfrac{1}{4} N g_M^2 f_L^2 \delta_{\alpha\beta}\,, \tag{8a}$$

$$M_{\mu\nu}^2 = \tfrac{1}{2} g_N^2 f_L^2 \, \mathrm{tr}\, T_\mu^{\mathrm{N}} T_\nu^{\mathrm{N}} = \tfrac{1}{4} M g_N^2 f_L^2 \delta_{\mu\nu}\,,$$

$$M_{\alpha\beta}^2 = M_{\mu\alpha}^2 = -\tfrac{1}{2} g_M g_N f_L^2 \, \mathrm{tr}(T_\mu^{\mathrm{N}} \Sigma T_\alpha^{\mathrm{M}} \Sigma^\dagger)\,, \tag{8b}$$

where g_M (g_N) is the SU(M) (SU(N)) gauge coupling. Note that the sum of the gauge boson mass-squared, $\mathrm{tr}\, M^2$ from (8a), is independent of Σ which is why the order g^2 potential does not depend on the vacuum alignment. All of the Σ dependence is in the mixing term (8b).

The contribution from gauge boson loops to the Coleman–Weinberg effective potential is

$$(3/64\pi^2)\, \mathrm{tr}\, M^4 \ln(M^2/\mu^2)\,, \tag{9}$$

where μ is a renormalization scale. A change of the value of μ can be compensated by a corresponding change in the coefficients of the order g^4 terms in the potential, which we cannot compute. But we can extract from (9) the nonanalytic term

$$[3 \ln(g^2)/64\pi^2]\ \mathrm{tr}\, M^4\,, \tag{10}$$

where g^2 is a typical gauge coupling. This term will dominate for small g^2. For small g^2, $\ln g^2$ is negative and the minimum of (9) is obtained when $\mathrm{tr}\, M^4$ is maximized. But we are interested in the strong coupling regime in which $\ln g^2$ is positive. It is not at all clear that (10) is relevant in this regime because higher order terms in g^2 may be even more important, but if we naively minimize (10) for $\ln g^2 > 0$, we find that the minimum occurs when the term (8b) vanishes, which happens for

$$\Sigma = I \quad \text{(or equivalent)}\,. \tag{11}$$

For $\Sigma = I$, both the gauge symmetries are spontaneously broken, but the global symmetries combine with the broken gauge symmetries to leave an unbroken global su(M) × su(N) symmetry. The unbroken su(M) is generated by the sum of the initial global su(M) and the gauge SU(M) generators. The unbroken su(N) is generated by the initial global su(N) generators minus the complex conjugates of the gauge SU(N) generators. It is evident from (5) that these combinations are unbroken if $\Sigma = I$.

If this symmetry breaking pattern is realized, then there is an su(L) × su(M) × su(N) global symmetry remaining in the low energy theory below the confinement scale. The LH L type fermions, which transform as an L under global su(L) and as an $(M, \bar{N})$ under the gauged SU(M) × SU(N) symmetries, transform under the unbroken global su(L) × su(M) × su(N) as (L, M, N). Thus they remain massless and saturate the anomaly conditions, exactly like the massless fermions in the confining picture.

Clearly, if the vacuum alignment is given by (11), the structure of the physics at low energies is the same no matter which of the three gauge groups is regarded as strong and confining. In each case, there is a massless (L, M, N) of LH fermions. But if the vacuum alignment goes the other way, as it does when two of the gauge interactions are weak, then the realization of the flavor symmetry depends on which interaction is strong. Suppose, for example, that $L = M = N$. If the SU(L) interaction becomes strong at a renormalization scale at which the other two interactions are still weak, then the SU(M) and SU(N) gauge groups align to leave the diagonal SU(M) unbroken, and the global su(M) × su(N) global symmetry breaks down to the diagonal su(M) [‡4]. However, if instead the SU(N) interaction, say, is strong, then it is the su(L) × su(M) global symmetry which is broken to a diagonal subgroup. We see that the theory has at least three distinct phases. It seems quite plausible that, if all three interactions get strong at roughly the same scale, then there is a fourth phase in which the gauge interactions can be treated symmetrically; in this phase, (11) is satisfied and massless composite fermions are produced.

Thus, in this model, it is reasonable to suppose that the global chiral symmetries do not break and that massless composite fermions result. In a separate note [9], we will argue that it may be possible to extend structures of this kind to produce models in which the composite fermions have nontrivial Yukawa couplings to composite Higgs induced by weak gauge couplings.

[‡4] If the remaining gauge symmetry is still asymptotically free, we expect the SU(L) global symmetry to break down to SO(L) (see ref. [11]).

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