NEUTRINO MASSES AND FAMILY SYMMETRY ☆

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Neutrino masses in the 100 eV–1 MeV range are permitted if there is a spontaneously broken global family symmetry that allows the heavy neutrinos to decay by Goldstone boson emission with a cosmologically acceptable lifetime. The family symmetry may be either abelian or nonabelian; we present models illustrating both possibilities. If the family symmetry is nonabelian, then the decay $\tau \to \mu$ + Goldstone boson or $\tau \to e$ + Goldstone boson may have an observable rate.

In the standard model of the electroweak interactions with minimal fermion and Higgs field content, the SU(2) × U(1) gauge symmetry prevents neutrino masses from being generated by couplings of renormalizable type. Neutrinos need not be exactly massless, however. New physics associated with an energy scale M well above the scale of electroweak symmetry breaking may give rise to effective nonrenormalizable couplings of the form

$$(g_{ab}/M)(L_a H)(L_b H) . \tag{1}$$

Here $a, b = e, \mu, \tau$ are family indices, the g_{ab} are coupling constants, H is the Higgs doublet, and the L_a are the lepton doublets expressed in the basis in which the charged leptons e^-, μ^-, τ^- are mass eigenstates. (The repeated family indices are summed.) When the Higgs doublet acquires the vacuum expectation value

$$\langle H \rangle = \begin{pmatrix} v \\ 0 \end{pmatrix} , \tag{2}$$

the couplings of eq. (1) generate the neutrino mass matrix

$$(v^2/M) g_{ab} \nu_a \nu_b \, . \tag{3}$$

Neturino masses are of great intrinsic interest because they reveal features of the new physics at mass scale M, physics which is currently inaccessible to conventional accelerator experiments.

The neutrino mass matrix is severely constrained by experiment [1]. Limits on neutrinoless double beta decay require the ee element to be less than about 10 eV, and there must be an eigenstate, predominantly ν_e, with mass less than about 40 eV. There are also very good limits from accelerator experiments [2] on the mixing of ν_μ with ν_e and ν_τ, and less stringent limits from reactor experiments [3] on the mixing of ν_e with ν_τ.

It was recently stressed by Dugan et al. and by Shanker [4] that all of these experimental constraints can be reconciled with the existence of a relatively heavy ($\gg 10$ eV) neutrino which mixes significantly with ν_e, provided that the mass matrix respects a U(1) global symmetry. This symmetry may be either $U(1)_{e-\mu+\tau}$ (electron number minus muon number plus tau number) or $U(1)_{e+\mu-\tau}$ (electron number plus muon number minus tau number). In the first case the most general allowed neutrino mass term is

$$m_\nu \nu_\mu (\cos\vartheta \, \nu_\tau + \sin\vartheta \, \nu_e) , \tag{4a}$$

ν_μ acquires a Dirac mass m_ν by pairing up with a linear combination of ν_τ and ν_e, while the ortho-

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gonal linear combination $-\sin\vartheta\,\nu_\tau + \cos\vartheta\,\nu_e$ remains exactly massless. Thus, the weak-interaction eigenstate ν_μ is also a mass eigenstate, and limits [3] on ν_e–ν_τ mixing require $\sin^2 2\vartheta \leqslant 0.16$. (For some values of m_ν, a better limit on $\sin^2 2\vartheta$ can be obtained from ^{3}H and ^{64}Cu beta decay experiments [1].) In the second case, the mass term is

$$m_\nu \nu_\tau (\cos\vartheta\,\nu_\mu + \sin\vartheta\,\nu_e)\,, \qquad (4b)$$

now ν_τ is a mass eigenstate and the limits [2] on ν_e–ν_μ mixing require $\sin^2 2\vartheta \lesssim 0.006$. In both cases, m_ν is required to be less than about 0.5 MeV, the experimental limit on the ν_μ mass. If m_ν is large ($\gtrsim$ 1 keV), then even very small U(1)-violating entries in the neutrino mass matrix tend to induce unacceptably large ν_μ–ν_τ mixing. It is sensible, therefore, to assume that the global U(1) symmetry is exact.

The properties of massive neutrinos are also sharply constrained by cosmological considerations. A (four-component) neutrino with mass greater than 50 eV must decay in the lifetime of the universe [5]; otherwise the mass density due to neutrinos today would exceed the observational bound. A neutrino much heavier than 50 eV is required to decay quite early. Of course, the energy density of the decay products of the neutrino must be acceptably small today. But, furthermore, if these decay products dominated the universe at earlier times, they must not have unduly delayed the growth of fluctuations in the energy density, which would have impeded the formation of large scale structure [6,7]. By demanding that fluctuations at a comoving scale of about $5h^{-1}$ Mpc were able to grow by at least a factor of $\lambda \times 10^3$ since horizon crossing, one derives the bound [7]

$$\tau \lesssim 10^2\,\mathrm{yr}(1\ \mathrm{keV}/m_\nu)^2(\Omega h^2)^{3/2}\lambda^{-3/2} \qquad (5)$$

on the lifetime τ of the neutrino, for $m_\nu \gtrsim 1$ keV. Here the Hubble constant $H_0 = 100\,h$ km/s/Mpc has been used, and Ω is the present energy density of the universe in units of the critical density.

What could cause the heavy neutrino to decay so rapidly? In the standard model, the radiative decay $\nu_{\mathrm{heavy}} \to \nu_{\mathrm{massless}} + \gamma$ is GIM-suppressed [8]; the rate is much too small for $m_\nu \lesssim 1$ MeV. In any case, a radiative decay consistent with eq. (5) can be ruled out by astrophysical considerations [9]. A more attractive possible explanation for the short lifetime is that the exact global U(1) symmetry under which the neutrinos transform is embedded in a larger global symmetry which is spontaneously broken at the large mass scale M, and that the heavy neutrino decays by emission of a Goldstone boson [10] g, $\nu_{\mathrm{heavy}} \to \nu_{\mathrm{massless}} + \mathrm{g}$. Thus the neutrino mass and decay mode both arise from the same physics. In this paper, we consider some simple models which illustrate this possibility.

It there are only three families, then the largest possible global symmetry under which the neutrinos can transform is U(3). But we can exclude the possibility that such a U(3) symmetry is spontaneously broken at the mass scale M. For if M is small enough for the neutrino lifetime to satisfy eq. (5), then the branching ratio for $\mu \to$ eg will be unacceptably large [10]. Assuming a neutrino decay rate

$$\Gamma(\nu_{\mathrm{heavy}} \to \nu_{\mathrm{massless}} + \mathrm{g}) \sim m_\nu^3/32\pi M^2\,, \qquad (6)$$

eq. (5), with the conservative choice $\Omega h^2 = \lambda = 1$, becomes

$$M \lesssim 7 \times 10^6\ \mathrm{GeV}(m_\nu/1\ \mathrm{keV})^{1/2}\,, \qquad (7)$$

or $M \lesssim 10^8$ GeV for $m_\nu \lesssim 0.5$ MeV. But then

$$\Gamma(\mu \to \mathrm{eg}) \sim 10^{-2}(10^8\ \mathrm{GeV}/M)^2\,\Gamma(\mu \to \mathrm{e}\nu\bar{\nu}) \qquad (8)$$

exceeds the experimental bound [11].

An acceptable model, then, must have a smaller global family symmetry, and a suppressed Goldstone boson coupling to e–μ. The e–e and γ–γ couplings of the Goldstone boson must also be suppressed; otherwise the Goldstone boson emission from stars will be excessive [12]. Both conditions can be satisfied in a model with an abelian family symmetry. To illustrate this possibility, we construct a model, resembling the model of ref. [13], with a $\mathrm{U}(1)_e \times \mathrm{U}(1)_\mu \times \mathrm{U}(1)_\tau$ global symmetry spontaneously broken to $\mathrm{U}(1)_{e-\mu+\tau}$.

In this model, electron number, muon number, and tau number are all exact symmetries. In addition to the standard left-handed lepton doublets and right-handed charged lepton singlets, we include four SU(3) × SU(2) × U(1) singlet fermions ν_e^c, $\nu_{\mu a}^c$, $a = 1, 2$, ν_τ^c; the $\mathrm{U}(1)_e \times \mathrm{U}(1)_\mu \times \mathrm{U}(1)_\tau$ charges of all the fermions are shown in table 1. The standard Higgs doublet is neutral under the global family symmetry, and there are also SU(3) × SU(2) × U(1)-singlet complex scalar fields $\varphi_{e\mu}$ and $\varphi_{\mu\tau}$ carrying

Table 1

Lepton field	$U(1)_e \times U(1)_\mu \times U(1)_\tau$ charge
L_e	(1, 0, 0)
e^c, ν_e^c	(−1, 0, 0)
L_μ	(0, 1, 0)
$\mu^c, \nu^c_{\mu,1}, \nu^c_{\mu,2}$	(0, −1, 0)
L_τ	(0, 0, 1)
τ^c, ν_τ^c	(0, 0, −1)

$U(1)_e \times U(1)_\mu \times U(1)_\tau$ charges (1, 1, 0) and (0, 1, 1) respectively. In this mode, the most general lepton Yukawa couplings consistent with the gauge and global symmetries are

$$-L_{\rm Yuk} = (m_e/v)(L_e H)e^c + (m_\mu/v)(L_\mu H)\mu^c + (m_\tau/v)(L_\tau H)\tau^c + g_e(L_e H^*)\nu_e^c + g_\mu^a(L_\mu H^*)\nu^c_{\mu,a} + g_\tau(L_\tau H^*)\nu_\tau^c + \lambda^a_{e\mu}\varphi_{e\mu}\nu_e^c\nu^c_{\mu,a} + \lambda^a_{\mu\tau}\varphi_{\mu\tau}\nu^c_{\mu,a}\nu^c_\tau + \text{h.c.} \tag{9}$$

We assume that $\varphi_{e\mu}$ and $\varphi_{\mu\tau}$ get vacuum expectation values at the mass scale M, breaking the $U(1)_e \times U(1)_\mu \times U(1)_\tau$ symmetry down to $U(1)_{e-\mu+\tau}$. All the right-handed neutrinos ν^c thus acquire masses of order λM. (We introduced two right-handed muon neutrinos in order to ensure this.) The tree diagrams of fig. 1 then give rise to operators of the form eq. (1), which, upon the spontaneous breakdown of the electroweak gauge symmetries, generate neutrino masses of the form eq. (4a) with

$$m_\nu \sin\vartheta \sim (v^2/M) g_e g_\mu/\lambda_{e\mu} , \tag{10a}$$

$$m_\nu \cos\vartheta \sim (v^2/M) g_\mu g_\tau/\lambda_{\mu\tau} . \tag{10b}$$

Associated with the two spontaneously broken U(1) symmetries are two Goldstone bosons which allow the heavy neutrino to decay at a rate

$$\Gamma(\nu_{\rm heavy} \to \nu_{\rm massless} + g) \sim (m_\nu^5/32\pi M^2)\sin^2 2\vartheta , \tag{11}$$

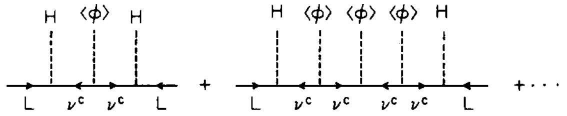

Fig. 1. Tree graphs generating the operator eq. (1).

suppressed by the mixing angle. The bound eq. (5) on the neutrino lifetime with the conservative choice of parameters $\lambda = \Omega h^2 = 1$ is satisfied provided

$$M \lesssim 7 \times 10^6\ \text{GeV} \sin 2\vartheta (m_\nu/1\ \text{keV})^{1/2} . \tag{12}$$

In this model, the existence of a neutrino in the mass range 1 keV–1 MeV which decays sufficiently rapidly implies that some Yukawa couplings are small; combining (10a) and (12) gives

$$g_e g_\mu/\lambda_{e\mu} \lesssim 10^{-4} \sin^2\vartheta (m_\nu/1\ \text{keV})^{3/2} . \tag{13}$$

In view of the small couplings already required by the charged lepton masses, Yukawa couplings satisfying (13) are not terribly implausible.

The Goldstone bosons have no direct coupling to electrons, and the coupling generated by loop graphs is negligibly small. Furthermore, because the spontaneously broken symmetries, acting on the charged leptons, are vectorlike, the Goldstone bosons have no coupling to two photons. Hence, the model is safe from the constraints [12] on M derived from stellar Goldstone boson emission.

The suppression of the $\mu \to$ eg rate and the e–e and γ–γ couplings of the Goldstone boson need not imply that the family symmetry is abelian. As an example of a model with a nonabelian family symmetry, we construct a model with global $SU(2) \times U(1) \times U(1)$ symmetry spontaneously broken to $U(1)_{e-\mu+\tau}$. In this model, τ and μ form a doublet under the global SU(2) symmetry, and the decay $\tau \to \mu g$ occurs at a rate which is consistent with current experimental limits but which is potentially observable.

In addition to the standard leptons, the model contains four gauge-singlet fermions transforming nontrivially under the global symmetry; the $SU(2) \times U(1) \times U(1)$ quantum numbers of all the leptons are

Table 2

Lepton field	$SU(2) \times U(1) \times U(1)$ charge
L_e	(1, 1, 0)
$e^c, \nu^c_{e,1}$	(1, −1, 0)
$\nu^c_{e,2}$	(1, 1, 2)
L_i	(2, 0, 1)
e_i^c, ν_i^c	(2, 0, −1)

listed in table 2. There is a Higgs doublet H that is neutral under the global family symmetry, plus Higgs doublets H^{ij} with quantum numbers (3, 0, 0) under SU(2) × U(1) × U(1). Additional gauge-singlet scalar fields η^i, χ^{ij}, and φ have global quantum numbers (2, 1, 1), (3, 0, 2), and (1, 0, −2) respectively.

We assume that the scalar fields η^i, χ^{ij} and φ acquire vacuum expectation values

$$\langle\eta^i\rangle \sim \begin{pmatrix} M \\ 0 \end{pmatrix}, \quad \langle\chi^{ij}\rangle \sim \begin{pmatrix} 0 & M \\ M & 0 \end{pmatrix}, \quad \langle\varphi\rangle \sim M\,, \qquad (14)$$

that break the SU(2) × U(1) × U(1) global symmetry to $U(1)_{e-\mu+\tau}$ at the scale M. The Higgs doublets acquire vacuum expectation values

$$\langle H\rangle \sim v, \quad \langle H^{ij}\rangle \sim \begin{pmatrix} 0 & v \\ v & 0 \end{pmatrix}, \qquad (15)$$

which break the SU(2) × U(1) gauge symmetry but preserve the $U(1)_{e-\mu+\tau}$ global symmetry. The SU(2) gauge indices are not displayed in eq. (15); it is assumed that the vacuum expectation values of all the Higgs doublets line up so that electromagnetism is unbroken. In fact, only one linear combination of the four Higgs doublets need remain light compared to the mass scale M [14].

The most general lepton Yukawa couplings consistent with the gauge and global symmetries are ‡1

$$-L_{Yuk} = g_e L_e H e^c + g_1 L_i H^{ij} e^c_j + g_2 \epsilon^{ij} L_i H e^c_j$$
$$+ h L_e H^* \nu^c_{e,1} + f_1 L_i H^{*ij} \nu^c_j + f_2 \epsilon^{ij} L_i H^* \nu^c_j$$
$$+ \lambda_1 \nu^c_{e,1} \nu^c_i \eta^i + \lambda_2 \nu^c_{e,1} \nu^c_{e,2} \varphi + \lambda \nu^c_i \nu^c_j \chi^{ij} + \text{h.c.} \qquad (16)$$

The four right-handed neutrinos acquire masses of order λM from the vacuum expectation values of η^i, φ, and χ^{ij}, and tree diagrams generate operators of the form eq. (1). The neutrino masses then have the form eq. (4a), if we identify $\nu_{1,2} = \nu_{\mu,\tau}$, with

$$m_\nu \sin\vartheta \sim (v^2/M) hf/\lambda\,, \qquad (17a)$$

$$m_\nu \cos\vartheta \sim (v^2/M) f^2/\lambda\,. \qquad (17b)$$

The masses $m_{e,\mu,\tau}$ of the charged leptons are independent free parameters ‡2.

Associated with the symmetry breakdown are four Goldstone bosons, two of which allow the heavy neutrino to decay at the rate given by eq. (6), unsuppressed by the mixing angle. The process $\tau \to \mu g$ can also occur, at a rate

$$\Gamma(\tau \to \mu g) \sim (10^6\ \text{GeV}/M)^2 \Gamma(\tau \to \mu\nu\bar{\nu})\,. \qquad (18)$$

Thus, for values of M satisfying the cosmological bound eq. (7), the rate for $\tau \to \mu g$ is potentially observable. The muons produced by this decay mode can be distinguished from those produced in $\tau \to \mu\nu\bar{\nu}$, because the muons have a fixed energy in the tau rest frame, and they are unpolarized.

We have not considered bounds on the scale M which might apply if the Goldstone bosons coupled to quarks from, for example, the experimental limit on the decay $K \to \pi g$. There is no reason for the quarks to transform under the global symmetry of the leptons. Even in a grand unified model, the family symmetry can be spontaneously broken at the unification scale, presumably much larger than M, to a global symmetry under which only the leptons transform.

In the examples discussed here, the unbroken global symmetry was chosen to be $U(1)_{e-\mu+\tau}$. The models can be trivially modified so that the unbroken symmetry is $U(1)_{e+\mu-\tau}$; in the second model, the tau decay mode can then be chosen to be either $\tau \to \mu g$ or $\tau \to eg$ ‡3.

Although either the $U(1)_{e-\mu+\tau}$ or the $U(1)_{e+\mu-\tau}$ global symmetry forbids neutrinoless double beta decay, neither forbids ordinary beta decay. Simpson [15] has recently reported evidence in tritium beta decay for a neutrino of mass $m_\nu = 17$ keV which mixes with the electron neutrino with mixing angle $\sin^2\vartheta \sim 0.03$. This result, together with limits on ν_μ–ν_e mixing, suggests that the neutrino mass matrix respects a $U(1)_{e-\mu+\tau}$ symmetry. Thus, Simpson's discovery, if correct, provides revealing information about the family symmetry of the leptons. The con-

‡1 These couplings respect an additional U(1) symmetry under which only φ and $\nu^c_{e,2}$ transform. This accidental symmetry need not be respected by the Higgs potential. If it is, it too is spontaneously broken.

‡2 In terms of the Yukawa couplings, these masses are $m_e \sim g_e v$, $m_\mu \sim (g_1 - g_2)v$, $m_\tau \sim (g_1 + g_2)v$.

‡3 The model with the decay mode $\tau \to eg$ is less attractive, because the electron mass is proportional to a difference of two Yukawa couplings, which must be tuned to one part in a thousand.

siderations of this paper suggest that interesting information about family symmetries might be obtained in a search for the decay modes $\tau \to e$ + Goldstone boson and $\tau \to \mu$ + Goldstone boson.

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