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GAUGE THEORIES IN THE MANY-GLUON LIMIT*

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We argue that the idea that the dynamics of a gauge theory simplifies in the limit $N \to \infty$, where N is the number of colors, can be invoked even if the gauge group is an exceptional Lie group, rather than one of the classical groups. We also point out that quantum tunneling phenomena can in some cases survive in the $N \to \infty$ limit, contrary to the usual claim that the $N \to \infty$ limit is "classical."

I. INTRODUCTION

In this talk, we discuss two ideas about the behavior of gauge field theories with large gauge groups.

The first idea concerns the range of applicability of the insight[1] that the dynamics of a gauge theory simplifies in the $N \to \infty$ limit, where N is the number of colors. We argue that this insight can be usefully applied to theories with a gauge group that is an exceptional group rather than one of the classical groups. In the case of exceptional gauge groups, there is no systematic expansion in a parameter analogous to $1/N$. But we expect the nonperturbative dynamics of, for example, a strongly-coupled E_8 gauge theory to have certain characteristic qualitative features that are a consequence of the large number of gluon species in the theory.

The second idea concerns the nature of quantum fluctuations in the $N \to \infty$ limit. We argue that, contrary to the usual assumption, it may in some cases be misleading to regard the $N \to \infty$ limit of a gauge theory as a "classical" limit in which fluctuations are suppressed. Specifically, some theories respect a classical $U(1)_A$ symmetry that is broken by an anomaly to a discrete symmetry Z_k where k is of order N. We argue that the realization of this Z_k symmetry is influenced by quantum tunneling phenomena that do not freeze out as $N \to \infty$. We speculate that such fluctuations may, for $N \to \infty$, give rise to a light excitation associated with the spontaneous breakdown of the Z_k symmetry.

The motivation for addressing these issues arose in part from our desire to understand the behavior of the "hidden sector" of the low-energy superstring theory. It has been suggested[2]

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that the hidden sector is a strongly-coupled E_8 gauge theory. In characterizing how this theory behaves, it is very helpful to identify a relevant small parameter. We propose that d_G^{-1} is such a parameter, where $d_G = 248$ is the dimension of the group, equal to the number of gluon species.

In section 2, we describe how the behavior of a gauge theory in the limit $d_G \to \infty$ can be analyzed, irrespective of the gauge group, by counting powers of d_G in Feynman diagrams. This analysis is sufficiently general to be applied to exceptional gauge groups as well as the classical groups. In section 3, we explain how fluctuation phenomena in the $N \to \infty$ limit can affect the realization of a discrete Z_k symmetry, where k is order N. For definiteness, we choose to discuss the particular case of pure supersymmetric Yang–Mills theory with gauge group $G = SU(N)$; this theory has a Z_N symmetry.

(The talk presented at the conference was mainly a review of Ref. 3. For the proceedings, we have chosen to expand on the concluding portion of the talk.)

2. THE MANY-GLUON LIMIT

A useful approach[1] to the nonperturbative behavior of quantum chromodynamics (QCD) is the expansion in $1/N$, where N is the number of colors. If color confinement is assumed to apply in the $N \to \infty$ limit, then a surprising number of *qualitative* features of meson phenomenology can be deduced[4–6] in this approach, such as the existence of many narrow resonances, the absence of exotic states, the OZI rule, and spontaneous breakdown of chiral symmetry[7]. Much can also be inferred about baryon physics in the $N \to \infty$ limit[5,8].

A $1/N$ expansion may be formulated for gauge theories other than QCD. QCD is a theory of SU(N) gauge fields coupled to fermions that transform as the defining representation of SU(N). We may consider changing the gauge group or changing the fermion representation. An expansion in $1/N$ can be carried out in an SO(N) or Sp($2N$) gauge theory[9], or in an SU(N) gauge theory with matter that transforms as the adjoint representation or a two-index tensor representation of the gauge group[3].

It has not yet proved possible to perform *quantitative* calculations in four-dimensional gauge theories by applying the $1/N$ expansion, but it may not be totally unrealistic to hope that this can someday be achieved. This hope is encouraged by the expectation that the $N \to \infty$ limit of, for example, QCD is remarkably simple. Indeed, assuming color confinement, it can be shown that in the $N \to \infty$ limit, QCD becomes a theory of an infinite number of *noninteracting zero-width* resonances[4–6]. (The resonances are glueballs and mesons; since baryon masses diverge linearly with N, baryons can be safely ignored in a discussion of the $N \to \infty$ limit of meson physics at fixed energy.) The $N \to \infty$ limit is simple enough that one can at least conceive of solving the theory explicitly in this limit. An expansion in powers of $1/N$ might then become tractable.

Even without the capability to do quantitative calculations, one can infer interesting qualitative features of meson physics by considering the properties of the $N \to \infty$ limit. Furthermore, these inferences are based on quite elementary considerations. The $1/N$ expansion is usually formulated[1] by assigning to each Feynman diagram a power of N determined by the topology of the diagram. The leading diagrams in the $N \to \infty$ limit are *planar*; they contain no quark loops and can be drawn on a sphere without any crossing of gluon lines. Each quark loop is suppressed by a factor of $1/N$, and each nonplanar gluon exchange is suppressed by a factor of $1/N^2$.

In fact, all of the insights that have been obtained into the properties of mesons in the $N \to \infty$ limit can be understood without making any explicit reference to the topology of the leading diagrams. What is crucial instead is the scaling behavior of the connected gauge-invariant Green functions of the theory as N becomes large. And this scaling behavior is a simple consequence of the color degeneracy of the gluons and quarks. That is, the tendency of the dynamics of QCD (and other gauge theories) to simplify in the $N \to \infty$ limit arises merely because the number of gluon species is of order N^2, and the number of quark species is of order N.

It is a valuable insight that the dynamics of a gauge theory with gauge group SU(N), SO(N), or Sp($2N$) simplifies dramatically when the number of gluon species is large. The point we wish to stress here is that this insight can be usefully applied to theories with a gauge group that is an exceptional Lie group rather than one of the classical groups. For exceptional gauge groups, we cannot formulate a systematic expansion in a parameter analogous to $1/N$, but we can nonetheless make qualitative statements about features of the dynamics that arise because the dimension d_G of the group is large. That a systematic expansion in powers of d_G^{-1} cannot be formulated for the exceptional groups is not so serious a handicap. Although such an expansion exists in principle for the classical groups, we do not know at present how to carry it out. The qualitative understanding of gauge theories with classical gauge groups that has been extracted by considering the $N \to \infty$ limit is all based on arguments that can be applied just as well to theories with exceptional gauge groups.

To justify the above remark, we may formulate a "many-gluon limit" of a gauge theory that makes no explicit reference to a specific gauge group. To see how this is possible, let us consider the simplest case—that of a pure Yang-Mills theory with a *simple* group G. The dimension of the group, and hence the number of gluon species, is denoted d_G. The quadratic Casimir invariant of the adjoint representation of G is denoted $C_2(G)$, and g is the (conventionally normalized) gauge coupling.

Now, the Feynman diagrams that can be constructed in this theory are the same for all G, but the diagrams carry group-theoretic weights that depend on G. How can we characterize this G-dependence? From relatively straightforward group-theoretic considerations, it can be shown that all the connected vacuum bubble diagrams with $m+1$ loops obey an inequality[10].

$$\begin{array}{c}\text{connected}\,(m+1)-\text{loop}\\ \text{vacuum diagram}\end{array} \quad < \quad A_m[g^2C_2(G)]^m \quad d_G \quad .$$

Here A_m is a numerical constant that is *independent* of the gauge group G. Furthermore, it is easy to construct examples of (planar) $(m+1)$–loop diagrams that carry the group theoretic weight $[g^2C_2(G)]^m d_G$. Thus, without specifying a particular gauge group, we may speak of a "many-gluon limit" in which d_G and $C_2(G)$ grow large with

$$\lambda = g^2C_2(G)$$

held fixed. In this limit, all of the connected vacuum bubble diagrams are at most of order d_G. Thus, the vacuum energy of this theory due to the zero-point fluctuations of the gluons is of order d_G. This behavior is a simple consequence of the d_G-fold degeneracy of the gluons.

One should note that, even though g^2 is really a running coupling constant, it is sensible to regard $\lambda = g^2C_2(G)$ as fixed as we change the size of the gauge group G. This makes sense because $C_2(G)$ can be scaled out of the renormalization group equation for λ when $C_2(G) >> 1$. Thus, although λ runs, it runs in a manner that is independent of the gauge group G when G is large, and we should take λ to be of order one as the size of G increases.

That the dynamics of the Yang–Mills theory simplifies as the gauge group G grows larger follows from the simple observation that the vacuum energy is of order d_G. The point is that the diagrams that contribute to the connected Green functions of gauge-invariant operators scale the same way with d_G as the connected vacuum bubbles do. For example, suppose that $B_1, \ldots, B_m$ are local operators, such as $tr(F_{\mu\nu}(x)F^{\mu\nu}(x))$, that are gauge-invariant and bilinear in the Yang–Mills field strength F; then,

$$\langle B_1 \ldots B_m \rangle_{\text{connected}} = O(d_G)\,.$$

It follows that

$$\frac{\langle B^2\rangle - \langle B\rangle^2}{\langle B\rangle^2} = O(d_G^{-1})$$

for any such gauge-invariant bilinear operator B. This result is readily generalized to higher dimension operators and nonlocal operators. We see, therefore, that the many-gluon limit $d_G \to \infty$ is a *classical* limit[11]; the root-mean-square fluctuations of the fields about their mean values are of order $d_G^{-1/2}$. The parameter $d_G^{-1/2}$ is the generalization for arbitrary gauge groups of the expansion parameter $1/N$ for the classical groups. This parameter characterizes how close the Yang–Mills theory is to the many-gluon limit.

We can go further if we make an assumption about the spectrum of the theory; namely, that the degeneracies in the spectrum are of order one, rather than, say, of order d_G. A strong motivation for this assumption that it is an expected consequence of color confinement. If all physical states are G-singlets, then there are no liberated gluons, and there is no reason to expect any degeneracy at all. But strictly speaking this assumption is logically independent of whether color is confined or not.

Making this assumption, we conclude that, since the Green function

$$\langle B_1 B_2 \rangle_{\text{connected}} = O(d_G)$$

has pole terms with residues of order d_G, the operator B couples to one-particle states with a strength of order $d_G^{1/2}$. (The possibility that we have excluded, by assumption, is that B couples to e.g., d_G degenerate one-particle states, each with a strength of order one.) Then,

$$\tilde{B} = d_G^{-1/2} B$$

is properly normalized to couple to a one-particle state. From

$$\left\langle \tilde{B}_1 \dots \tilde{B}_m \right\rangle_{\text{connected}} = O(d_G^{1-m/2}) \quad ,$$

we see that the m-body connected scattering amplitude is of order $d_G^{1-m/2}$. We learn, therefore, that in the many-gluon limit, Yang–Mills theory becomes a free-field theory. "Glueball" resonances have decay widths that go to zero like d_G^{-1} as $d_G \to \infty$, and scattering cross sections that are of order d_G^{-2}. There must, in fact, be an infinite number of these noninteracting zero-width resonances in the limit $d_G \to \infty$, in order that the Green functions behave at large momentum as predicted by renormalization-group-improved perturbation theory.

The above conclusions are familiar for an SU(N) gauge theory[4–6]. Our purpose here is just to emphasize their wider applicability. In, for example, an E_8 gauge theory ($d_G = 248$), the naive counting of the number of gluons suggests that the root-mean-square fluctuations of gauge-invariant operators about their mean values are of order $(248)^{1/2} \sim 1/16$, and that there are many narrow glueball resonances with widths of order $1/248$.

So far we have considered only pure Yang–Mills theory, but the analysis can easily be extended to gauge fields coupled to matter that transforms as the adjoint representation or the fundamental representation of the gauge group. To deal with the case of a theory containing "quarks" in the fundamental representation, we may derive an inequality of the form[10]

$$\begin{matrix}\text{connected}\,(m+1) - \text{loop vacuum} \\ \text{diagram with Quark loops}\end{matrix} \quad < \quad A'_m [g^2 C_2(G)]^m \, d_F \quad ,$$

where d_F is the dimension of the fundamental representation, the number of quark species.

The spectrum of this theory contains both glueball states and mesons that couple to gauge-invariant quark bilinear operators. By a straightforward generalization of our previous argument, it can be shown, assuming again that all degeneracies are of order one, that the connected ℓ-meson k-glueball scattering amplitude is of order $d_F^{1-\ell/2} d_G^{-k/2}$ (for $\ell \neq 0$). Thus, meson-glueball mixing is of order $(d_F/d_G)^{1/2}$ and meson widths are of order $1/d_F$.

In a gauge theory in which the gauge group is a classical group, there is just one expansion parameter $1/N$. But in a general theory of gauge fields coupled to matter in the fundamental representation, there are two small parameters analogous to $1/N$; the two parameters are $d_G^{-1/2}$ and d_F^{-1}. Qualitative conclusions concerning the physics of the theory can be inferred from the smallness of these parameters. It seems reasonable *a priori* to expect that these conclusions apply to a theory whose gauge group is any one of the exceptional Lie groups (with the possible exception of G_2).

3. SUPERSYMMETRIC YANG-MILLS THEORY

We turn now to a discussion of the possible relevance of fluctuation phenomena in the $N \to \infty$ theory. For the sake of definiteness, we will consider one example of a theory in which these phenomena can arise – pure supersymmetric Yang–Mills theory with gauge group $G = SU(N)$. This is a theory of gluons coupled to a single two-component spin-1/2 fermion field λ (the gluino) that transforms as the adjoint representation of the gauge group. It follows from the discussion in section 2 that a similar analysis ought to apply to supersymmetric Yang-Mills with any large gauge group. (Also, the effects we will describe can occur for other choices of the fermion representation content of theory.)

If color confinement holds, then in the $N \to \infty$ limit supersymmetric Yang–Mills theory becomes a theory of an infinite number of noninteracting zero-width resonances. The resonances are glueballs and mesons (gluino-gluino bound states) with order one mixing between glueballs and mesons. At finite N, the resonances acquire widths of order $1/N^2$, and two-body scattering cross sections of order $1/N^4$.

What can we say about the realization of the symmetries of the theory? Of particular interest is the realization of supersymmetry. It is known, however, that supersymmetry cannot be spontaneously broken in pure supersymmetric Yang–Mills theory[12], for any value of N.

What other symmetries does this theory have? The classical action is invariant under the (gluino number) $U(1)_A$ transformation

$$\lambda \longrightarrow e^{i\alpha}\lambda \,.$$

But quantum mechanical effects arising from the triangle anomaly destroy this symmetry[13]. However, a discrete subgroup of the $U(1)_A$ symmetry does survive. Because of the anomaly, a rotation of the phase of the gluino field is equivalent to a rotation of the vacuum angle θ,

$$\theta \longrightarrow \theta + 2N\alpha.$$

Since θ is a periodic variable defined modulo 2π, a rotation with

$$\alpha = \frac{\pi}{N} m, \qquad m = 0, 1, \ldots, N-1,$$

really does leave the physics invariant. Therefore, this theory has an exact Z_N symmetry. (The discrete symmetry is Z_N rather than Z_{2N} because a phase rotation by $\alpha = \pi$ is equivalent to a spatial rotation by 2π, which is embedded in a *continuous* symmetry group of the theory.)

How is the Z_N symmetry realized? There is a plausible mechanism by which it could be spontaneously broken. If a gluino condensate forms, that is, if a Lorentz-invariant, gauge-invariant operator bilinear in λ has a vacuum expectation value,

$$\langle \lambda\lambda \rangle \neq 0,$$

then the Z_N symmetry is completely broken. The arguments[12] that show that supersymmetry is manifest do not exclude the formation of such a gluino condensate.

There is a good physics motivation for investigating whether gluino condensation occurs. If the Yang–Mills supermultiplet is weakly coupled to an *axion* supermultiplet and the gluinos condense, then supersymmetry *is* spontaneously broken [2]. (The super-partner of the axion becomes the Goldstino.) It is quite possible that the breaking of supersymmetry observed in Nature arises in this way.

A number of arguments support the contention that a gluino condensate forms in pure supersymmetric Yang–Mills theory:

- Gluon exchange generates an attractive interaction between a pair of gluinos in the color-singlet channel. We may expect this attractive interaction to drive an instability toward the formation of a pair condensate. A similar instability is presumably responsible for spontaneous breakdown of chiral symmetry in QCD.
- In Yang–Mills theory coupled to not 1 but N_f adjoint fermions, rigorous inequalities[14] combined with 't Hooft's anomaly condition show that for $N_f \geq 3$ the $SU(N_f)$ chiral symmetry is spontaneously broken to the subgroup $SO(N_f)$ (assuming confinement). The pattern of symmetry breakdown suggests the formation of a gluino condensate

$$\langle \lambda_i \lambda_j \rangle \propto \delta_{ij},$$

where $i, j = 1, 2, \ldots, N_f$ are "flavor" indices. If gluinos condense for $N_f \geq 3$, it is reasonable to expect that they condense also for $N_f = 1$.

- Supersymmetry relates gluino condensation to gluon condensation, and one expects gluons to condense (e.g., as a consequence of the conformal anomaly) [15].
- It is known on topological grounds that the theory has N degenerate vacua, and one is tempted to identify these vacua[12] with the N possible values of $\langle \lambda\lambda \rangle$. (Actually, this counting works for $G = SU(N)$ or $Sp(2N)$, but fails for other gauge groups.)

All of these arguments are fairly persuasive, but none is totally convincing.

The $1/N$ expansion gives further support for the claim that gluinos condense. In the $N \to \infty$, fluctuations in the phase of the gluino bilinear become suppressed. If we write

$$\lambda\lambda = \rho\, e^{i\sigma},$$

where ρ and σ are real fields, then counting powers of $1/N$ in connected Green functions shows that the typical root-mean-square fluctuations of σ are of order $1/N$. Because these fluctuations are very weak, it seems plausible that the ground state of the theory would be a state with long-range-order, a gluino condensate.

The $1/N$ expansion also provides us with some interesting insights into the nature of the fluctuations of σ. We expect that the discrete Z_N symmetry is spontaneously broken for $N \to \infty$ (and probably for all values of N). But we might also expect a Z_N symmetry to become indistinguishable from a $U(1)$ symmetry in the limit $N \to \infty$. Ought there to be, then, a massless "spin wave" (Goldstone boson) excitation in the theory as $N \to \infty$?

As far as the realization of the Z_N symmetry is concerned, our supersymmetric Yang–Mills theory is analogous to a Z_N spin system ("clock model") in four dimensions. A Z_N spin system in four dimensions has two phases, a low-temperature ordered phase and a high-temperature disordered phase. For any finite value of N, the ordered phase has a mass gap; correlation functions approach their asymptotic values at large separation exponentially. But if we take the limit $N \to \infty$ with the temperature *fixed*, the mass gap in the ordered phase approaches zero. In this sense, the Z_N spin system behaves in the $N \to \infty$ limit like a U(1) spin system, which has a massless spin wave excitation in the ordered phase. This analogy therefore suggests that, as $N \to \infty$, the Z_N symmetry of supersymmetric Yang–Mills theory *does* behave like a U(1) symmetry, and that there is an exactly massless "Goldstone boson" in spite of the explicit breaking of the U(1) symmetry due to the anomaly. But we should not accept this conclusion too readily.

In a Z_N spin model in four dimensions, for any finite value of N, spin fluctuations freeze out at sufficiently low temperature. To characterize how important the fluctuations are, we may consider the abundance of (three-dimensional) domain "walls" across which the spin rotates by the angle $2\pi/N$. Since a misalignment of nearest neighbors by the angle $2\pi/N$ is suppressed by a Boltzman factor

$$\exp(-\beta/N^2),$$

such domain walls are rare for $\beta^{-1} << \frac{1}{N^2}$. But for $\beta^{-1} \gtrsim \frac{1}{N^2}$, domain walls become abundant, and long-wavelength spin fluctuations may occur. The effect of these fluctuations is to reduce the mass gap m to

$$m \sim \exp(-N^2/\beta) \quad .$$

This behavior of the mass gap in the ordered phase of the *three-dimensional* Z_N spin systems can be easily understood. The mass gap may be interpreted as the magnetic screening mass in a Z_N gauge system that is dual to the spin system[16], and is proportional to the density of magnetic monopoles in the dual gauge system[17]. We expect that the behavior of the mass gap as a function of β and N is similar in any dimension $d > 3$. Note that, at fixed β, the mass gap approaches zero very rapidly as N increases.

Now, the Z_N spin system that describes the gluino condensate in supersymmetric Yang–Mills theory becomes very "cold" as $N \to \infty$; quantum fluctuations freeze out in this limit. Is it so cold that fluctuations of the Z_N "spins" are unimportant? To answer this question, we must determine the cost in action density of a domain wall across which the Z_N spin rotates by $2\pi/N$. This action density is governed by an effective Lagrangian for the field σ, the phase of the gluino bilinear, that has the form

$$\mathcal{L} = N^2(\nabla\sigma)^2 + V(N\sigma)\,.$$

The N^2 in front of the kinetic term reflects the usual tendency for fluctuations to be suppressed at large N. The potential $V(N\sigma)$ arises from the explicit breaking of the $U(1)_A$ symmetry due to the anomaly. It is essentially the dependence of the vacuum energy of pure (nonsupersymmetric) Yang–Mills theory on the vacuum angle θ, and can be expressed as a function of $N\sigma$ because a rotation of $N\sigma$ is equivalent to a rotation of θ. It is important to realize that the potential $V(\theta)$ is of order one [18], rather than of order N^2 as one might have naively expected. As a result, N can be scaled out of the effective Lagrangian $\mathcal{L}$, and the action density of a domain wall across which σ rotates by $2\pi/N$ (θ rotates by 2π) is of order one. Therefore, the Z_N spin fluctuations do not freeze out in the limit $N \to \infty$. This is the "nonclassical" behavior referred to in the introduction. Although the quantum fluctuations of σ are suppressed as $N \to \infty$, the distance in σ between the neighboring minima of the potential $V(N\sigma)$ is shrinking as $N \to \infty$, and quantum tunneling between neighboring minima therefore persists even for large N.

While we have established that Z_N spin fluctuations occur even for $N \to \infty$, we have not shown that these fluctuations are sufficiently abundant to dramatically reduce the mass of the σ particle. Supersymmetric Yang–Mills theory in the $N \to \infty$ limit is similar to a Z_N spin system at temperature $\beta^{-1} \sim 1/N^2$; in the spin system, the abundance of the spin fluctuations and hence also the mass gap are exponentially sensitive to the value of N^2/β. The corresponding parameter in the supersymmetric Yang–Mills theory is a number of order one that we have not computed. If this parameter is small enough, spin fluctuations are rare and the σ field behaves "classically." The mass of the σ particle (and of its supersymmetric partner) is therefore determined to good accuracy by the curvature of the potential $V(\theta)$ at its minimum. But if this parameter is large enough, the σ field tunnels easily from one minimum of $V(N\sigma)$ to a neighboring minimum, and the mass of the σ particle may be many orders of magnitude below the "classical" estimate. One is tempted to speculate, then, that because of copious quantum fluctuations of the σ field that survive in the $N \to \infty$ limit, there is a very light particle in the spectrum of supersymmetric Yang–Mills particle, associated with the spontaneous breakdown of the discrete Z_N symmetry.

Of course, on the basis of naive counting of powers of $1/N$, one expects the σ particle to have a mass m_σ of order one. Strictly speaking, the above considerations do not change this expectation. Since N scales out of the effective Lagrangian for the σ field, m_σ presumably does approach a finite nonzero limit as $N \to \infty$. We are suggesting, though, that this limiting value of m_σ may be much smaller than the characteristic mass scale Λ of the theory. One should also note that the mixing of the σ particle with glueball states must be similarly suppressed. Since the σ particle has a derivative coupling to the gluino number current $J^\mu = \lambda^\dagger \gamma^\mu \lambda$, its coupling to $\partial_\mu J^\mu$ is proportional to m_σ^2. But because of the anomaly $\partial_\mu J^\mu$ is a gluon bilinear. Hence, the spin fluctuations that lower the mass of the σ particle must also reduce the mixing of the σ with glueballs below that naively expected.

If pure supersymmetric Yang–Mills theory with gauge group $G = SU(N)$ does contain a very light excitation, then it is reasonable to expect supersymmetric Yang–Mills theory with any large gauge group to have the same property. For example, the supersymmetric E_8 gauge theory has a Z_{30} discrete symmetry, and one might expect there to be a light excitation in the spectrum of the theory associated with the spontaneous breakdown of the Z_{30} symmetry.

In fact, a phenomenon similar to that discussed above might arise in any large N gauge theory with a discrete Z_k symmetry, where k is of order N. We could consider, for example, an $SU(N)$ theory with fermions that transform as a two-index symmetric or antisymmetric tensor. (Larger representations of $SU(N)$ are excluded if we wish to maintain asymptotic freedom for $N \to \infty$.) One case of particular interest is QCD (fermions in the fundamental representation) with a number N_f of flavors that is of order N. In this case, the $U(1)_A$ symmetry is broken by the anomaly to a Z_{N_f} symmetry, but the symmetry is not really discrete. Rather, the Z_{N_f} is embedded in the continuous $SU(N_f) \times SU(N_f) \times U(1)_V$ chiral symmetry group of the theory. Nonetheless, the spin fluctuations we have described would occur in this theory, and might have interesting dynamical effects.

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